Fourier Expansion of Mode Coupling for Higher-Order PMD Definition

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Abstract *Simulations indicate that Fourier coefficients of mode coupling and total DGD describe natural PMD much more accurately than a Taylor expansion of the PMD vector. This is important for optimal choice and control of PMD compensators.*

Introduction

Poole's definition of 1st-order polarization mode dispersion (PMD) [1] is undisputed. The principal states-of-polarization (PSPs) and their differential group delay (DGD) are found from the frequency dependence of output polarization [1] or the polarization-dependent small-signal intensity modulation transfer function [2].

Higher-order PMD is usually expressed by Foschini's truncated **Taylor expansion of the PMD vector (TEPV)** [3]. Heismann has calculated the Jones matrix of a given TEPV [4]. The true frequency-dependent trajectory of the PMD vector in the Stokes space would be described by sums of sinusoids with arguments that depend linearly on frequency. But sinusoids are not well approximated by a Taylor series. Inevitably, an unphysical, infinite DGD will be predicted far off the optical carrier frequency.

A competing approach is the **EMTY** method, an exponential expansion of the Jones matrix by **Eyal, Marshall, Tur and Yariv** [5].

A direct relation to physical fiber parameters exists neither for TEPV nor for EMTY.

Most PMD simulations are carried out by assuming a **sequence of cascaded DGD sections (SDGD)**, because it is widely accepted that an infinite number of randomly cascaded sections produces "natural" PMD. A finite SDGD can also be used for PMD description [2, 6]. It can be graphically displayed by a DGD profile ("reference" in Fig. 1) [2, 6], and it emulates what a real fiber typically does.

Here a **Fourier expansion of mode coupling (FEMC)** is proposed. It is similar to SDGD but avoids the discretization by a finite number of sections. Simulations indicate that it can describe natural PMD much more accurately than TEPV and EMTY.

Definition and DGD profiles

The direction change in the normalized Stokes space between adjacent DGD sections is the retardation of a mode converter between them. Seen by an observer who looks in the direction of the preceding DGD section this direction change occurs up/down or right/left. This amounts to discrete in-phase or quadrature mode coupling between local PSPs. If the number of sections in a SDGD approaches infinity, mode coupling becomes continuous. An FEMC can now describe PMD as follows:

- A frequency-independent mode conversion at the fiber input. This is described by 2 parameters, for example a retardation and an orientation.
- A total DGD of the DGD profile.
- A frequency-independent elliptical retarder (3 parameters) at the output of the medium.
- Complex spatial Fourier coefficients F_k of mode coupling [6] along the birefringent medium. The above-mentioned total DGD is that which can be measured when mode coupling is removed and the DGD profile is straightened out.

The first 3 items simply describe 1st-order PMD, and the 4th adds mode coupling to complete a higher-order FEMC. Mode coupling bends the DGD profile [6]. Bends at discrete positions would correspond to a SDGD. The FEMC Fourier coefficients describe DGD profile bending in a continuous manner (Fig. 1).

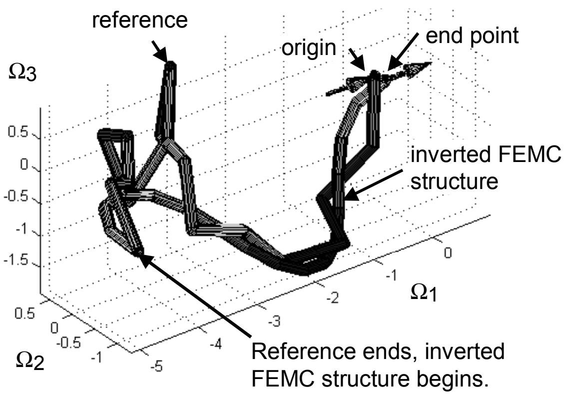

Fig. 1: DGD profile of reference (exemplary PMD medium) cascaded with inverted FEMC structure (which thereby forms a PMD equalizer). Scaling unit is 1 DGD section length of the reference structure.

No mode coupling occurs in the 1st-order case. In a 2nd-order FEMC, the zero-order coefficient F_0 is specified by 2 real parameters and coils the DGD profile. Whether coiling occurs up/down or right/left or in a mix of these cases depends on the phase angle of F_0. The coiling radius is inversely proportional to $|F_0|$. Other F_k will wind spirals if they occur alone. F_k combined with F_{-k} can result in a forth-and-back bending of the DGD profile. Each higher order of FEMC requires two extra Fourier coefficients $F_{\pm k}$, which amounts to 4 more real parameters.

The required number of real parameters is listed in Table 1. All methods need 3 more parameters to specify a frequency-independent elliptical retarder at the fiber output. For SDGD the method order tells how many DGD sections there are.

Table 1: Number of real parameters

Method (below) and its order (right)	1	2	3	4
TEPV and EMTY		6	9	12
SDGD	3	5	7	9
FEMC		5	9	13

Cross polarization suppression

We give an FEMC example for method order 3 ($|k| \leq 1$). A random PMD medium has been taken as a reference. It is composed of 16 DGD sections with equal lengths. The length of one DGD section defines the normalized unit length in Fig. 1. The 1st-order PMD vector is $[-4.98, -1.24, -0.42]^T$, the DGD is 5.1 units. The reference is cascaded with a smoother DGD profile that is an inversion of the FEMC structure. It follows the reference profile with gentle bends and more or less cuts through the “messy” left part of the reference. For convenience the FEMC structure was represented by 16 sections (instead of an infinite number) of equal but variable length.

The FEMC coefficients were determined as follows: A Gaussian input pulse was assumed, with a width equal to the total DGD of the DGD profile used in the FEMC (assuming no mode conversion). This is not the only possible pulse shape and duration. But it makes sense to chose the total DGD rather than the 1st-order DGD because the former is related to the overall complexity of the PMD situation while the latter may even vanish. Pulse width and the identical total DGD were varied during the optimization. The PMD medium (reference) and the inverse of the structure defined by the FEMC were concatenated. The various parameters were adjusted so that the output signal was – as far as possible – in only one (co-)polarization mode, and that the impulse in the other (cross-)polarization had its residual amplitude maximum near the time origin – not elsewhere like in the case of 1st-order PMD. Fig. 2 shows the magnitudes of the electric fields in co- and cross-polarized output pulses. The unwanted polarization is ≥37.2 dB down.

For comparison, the TEPV was calculated up to 3rd order from the Jones matrix of the reference (PMD medium). Then the Jones matrix corresponding to this truncated TEPV was built [4]. The inverse of that matrix was cascaded with the reference. The same was also done for EMTY. For all methods the input pulse width was chosen identical to that after convergence of the FEMC. Table 2 shows orthogonal polarization suppression vs. method and its order. The FEMC holds an advantage over the TEPV and the EMTY method. The extinction improvement in dB by the addition of higher-order terms is several times larger for FEMC than for TEPV and EMTY. In a few more tested PMD examples the FEMC also held an advantage. This is not surprising, because FEMC and SDGD models are closely related to natural PMD.

The simulations indicate that a PMD compensator (PMDC) that could, for example, compensate the TEPV PMD orders 1 to 3 (cf. [7]) may not be the most efficient equalizer. Rather, a distributed PMDC [2, 6] could be preferable, with as sharp as possible a polarization transformation at its input, and controlled amounts of mode coupling along its length, for example defined by Fourier coefficients.

Note that for the calculations of Table 2, 3 more parameters were chosen than shown in Table 1, because the “equalizer” needed to be aligned to the reference to separate the output polarizations.

A drawback of FEMC is that an analytical solution is not known. Finding a straightforward solution might help in the control of distributed PMDCs – this is a question of not getting trapped in local optima during the PMD control process.

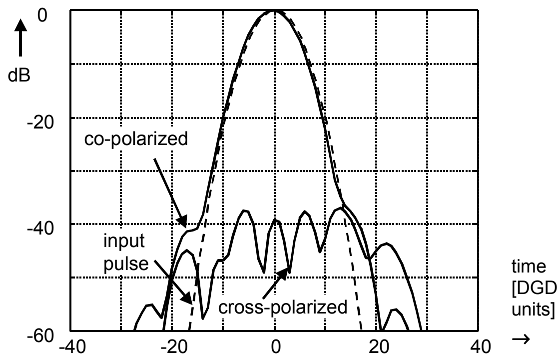

Fig. 2: Magnitudes of Gaussian input pulse, and of output pulses resulting from cascaded reference and inverted FEMC structure (order 3). Time scale is the same as in Fig. 1. Input pulse width is 8.1 DGD units.

Table 2: Suppression of cross polarization, using example of Fig. 1. Bold-faced case is plotted in Fig. 2.

Method order	1	2	3
Input pulse width and total DGD [DGD units]	5.1	5.3	8.1
TEPV		–15.7 dB	–15.2 dB
EMTY	–9.6 dB	–11.6 dB	–16.2 dB
FEMC		–21.3 dB	**–37.2 dB**

Conclusion

The proposed Fourier expansion of mode coupling seems to describe natural PMD much more accurately than competing higher-order PMD description methods. This indicates that distributed PMD compensators are more efficient than other types. Further work should concentrate on an efficient search of the FEMC coefficients.

References

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