Power Electronics

02.09.2015

Last Name:	Student Number:
First Name:	
Study Program:	☐ Professional Examination ☐ Performance Proof

Task: (Credits)	1 (25)	2 (25)	3 (25)	4 (25)		Total (100)		Mark

Duration: 120 minutes

Permitted auxiliaries:

- Nonprogrammable calculator without graphic display
- Drawing material (circle, triangle, ruler, pens…)

Please note the following remarks:

- You may only attend the exam if you have registered for it in the system PAUL. If you participate without registration your generated results will not be evaluated and accepted.
- Please hold your student identity card with photograph in readiness!
- Please label every blank sheet of paper with your name and your student number. For every task please use a new blank sheet of paper. Please do not use pencils and red pens.
- Every numerical calculation has to be furnished with units. Non-compliance will lead to point deduction.
- All approaches have to be documented in a comprehensible way! Specifying a single value as final result without a recognizable approach will not be evaluated.

Good Luck!

Task 1: Two-Quadrant Converter (25 Credits)

For an automotive application a two-quadrant converter is used. The output capacitor C is very large so that the output voltage ripple can be neglected. The technical specification of the converter is given below. First, all components are assumed to be ideal. Please assume steady state for the complete task.

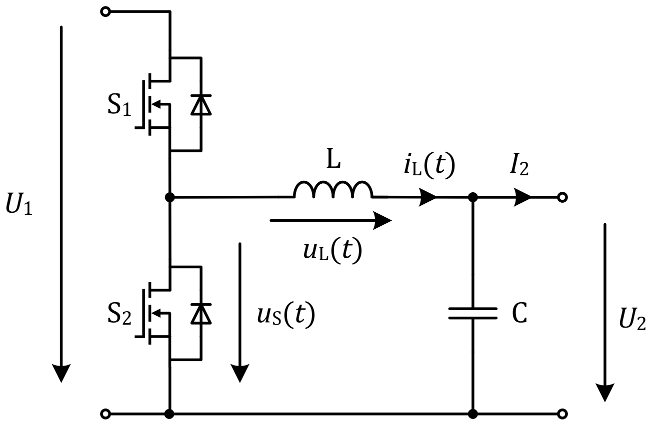


Figure 1.1: Circuit diagram of two-quadrant converter

- Input voltage: $U_1 = 48\ \mathrm{V}$
- Output voltage: $U_2 = 14\ \mathrm{V}$
- Inductance: $L = 10\ \mu\mathrm{H}$
- Switching frequency $f_S = 100\ \mathrm{kHz}$

1.1 Sketch the waveform of the voltages $u_S(t)$, $u_L(t)$ and the current $i_L(t)$ in the figure below. Make use of the marked initial value $i_L(t = 0)$. Indicate the amplitude of the voltages $u_S(t)$ and $u_L(t)$.

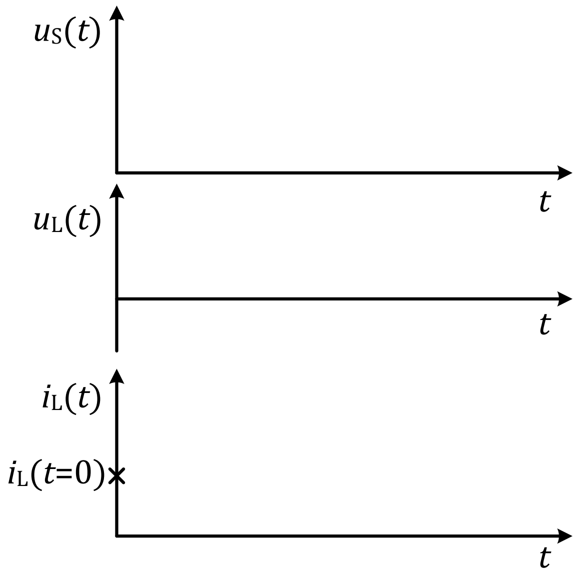


Figure 1.2: Diagrams for voltages and inductor current

1.2 Calculate the duty cycle $D = T_{\mathrm{on}}/T_{\mathrm{S}}$ which results in the desired output voltage U_2. (T_{on} is the on-time of S_1).

1.3 Calculate the current ripple Δi_{L} (peak-to-peak) for the given inductance.

1.4 Calculate the RMS-value I_{L} of the inductor current i_{L} for $I_2 = 0\ \mathrm{A}$.

1.5 Calculate the RMS-value I_{L} of the inductor current i_{L} for $I_2 = 10\ \mathrm{A}$.

Now it is assumed that the inductor has a serial resistance of $R_{\mathrm{L}} = 10\ \mathrm{m\Omega}$. Further it can be assumed that this small resistance does not influence the current waveform calculated before.

1.6 Calculate the resistance losses for $I_2 = 0\ \mathrm{A}$ and $I_2 = 10\ \mathrm{A}$.

1.7 Calculate the efficiency of the converter for $I_2 = 0\ \mathrm{A}$ and $I_2 = 10\ \mathrm{A}$.

Task 2: Boost Converter (25 Credits)

A boost converter supplies a resistive load which is connected to the output of the converter shown in the figure below.

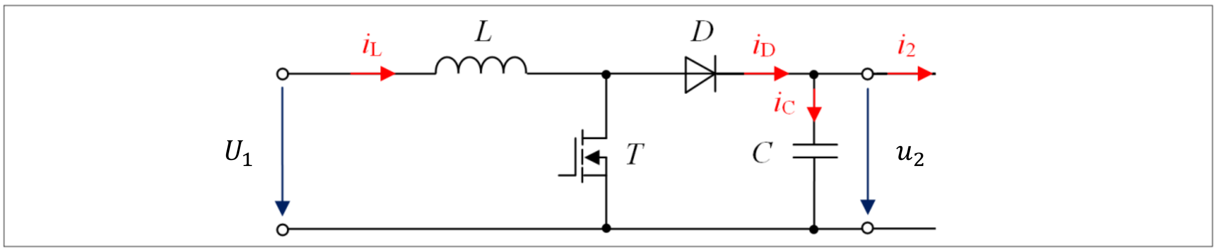


Figure 2.1: Topology of boost converter

In rated operation the following values were measured:

- Output voltage: $\bar{u}_2 = 400\ \mathrm{V}$
- Output power: $P_2 = 1800\ \mathrm{W}$
- Averaged input current: $\bar{i}_L = 15\ \mathrm{A}$
- Switching frequency $f_S = 10\ \mathrm{kHz}$

The boost converter can be considered as ideal and operates in steady state and continuous conduction mode (CCM). The input DC voltage U_1 is assumed constant. The output voltage ripple Δu_2 can be at first neglected.

2.1 Calculate the output current $\bar{i}_2$ and the value of the load resistor R_L. What can be said about the ripple of i_2?

2.2 Determine the value of the input voltage U_1 and the resulting duty cycle D.

2.3 The value of the inductor L is 1.68 mH. Calculate the current ripple Δi_L and draw the currents $i_L(t)$, $\bar{i}_L(t)$, $i_D(t)$ and $\bar{i}_2$ over two switching periods in the diagram of Figure 2.2.

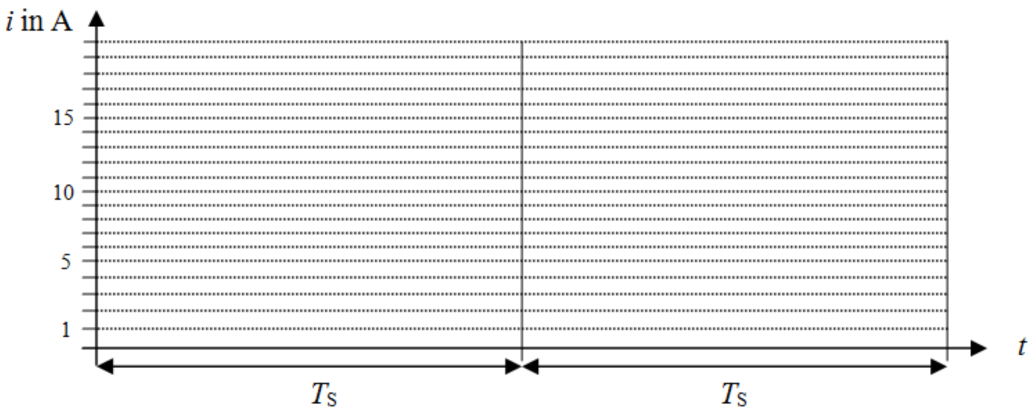


Figure 2.2: Inductor, diode and output current

2.4 Assume that duty cycle and input voltage do not change. Determine a new value of the load resistor R'_L so that the converter operates in BCM (boundary conduction mode).

2.5 The boost converter is still loaded with R_L. The output voltage ripple Δu_2 (peak-to-peak) was assumed to be negligible but a precise measurement revealed 0.2 V. Calculate the value of the output capacitor C and draw the voltage $u_2(t)$ over two switching periods in the diagram in Figure 2.3.

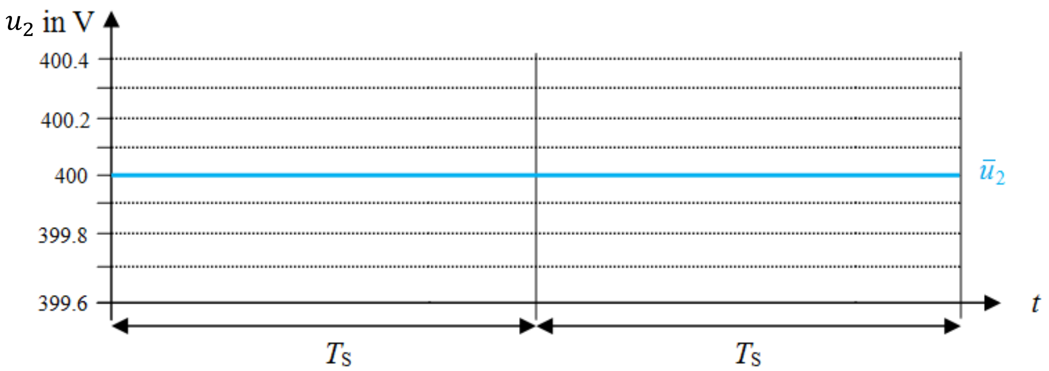


Figure 2.3: output voltage

2.6 Modify the converter of Figure 2.1 resulting in a PFC-rectifier so that an AC input ($U_0 = 230\ \mathrm{V}, f_\mathrm{grid} = 50\ \mathrm{Hz}$) can be used as supply. The output voltage is still $\overline{u}_2 = 400\ \mathrm{V}$.

Task 3: Four-Quadrant Converter (25 Credits)

A suburban train is equipped with a four-quadrant converter (4QC) to rectify the AC line voltage and to provide a nearly constant DC voltage for the traction drives and the on-board electrical systems. The system is shown in Figure 3.1:

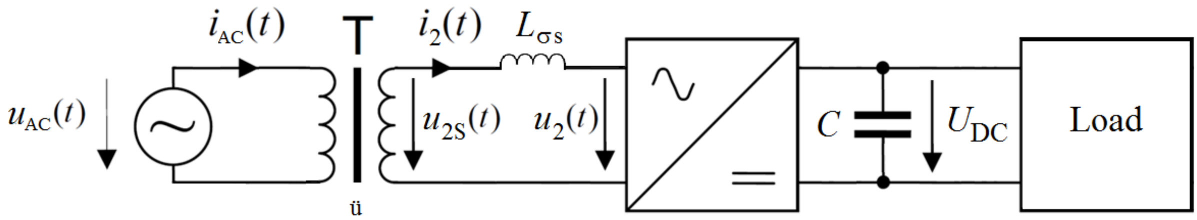


Figure 3.1: Transformer, four-quadrant converter and DC-link between grid and load

The following physical quantities are known:

- Grid voltage: $U_{AC} = 15\ \mathrm{kV}$
- Grid frequency: $f = 16.667\ \mathrm{Hz}$
- Transformer ratio: $\ddot{u} = 12$
- DC-link voltage: $U_{DC} = 2\ \mathrm{kV}$
- Switching frequency: $f_S = 300\ \mathrm{Hz}$

The four-quadrant converter can be considered as ideal (no losses, no interlocking times). The primary leakage inductance and the winding resistances of the transformer can be neglected. In the above diagram $L_{\sigma s}$ is the transformer's leakage inductance. So, the transformer circuit symbol is considered as ideal transformer.

3.1 Draw the structure of the PWM unit for interleaved switching mode in Figure 3.2. Input is the normalized reference voltage, output are the two switching signals for the two branches of the 4QC.

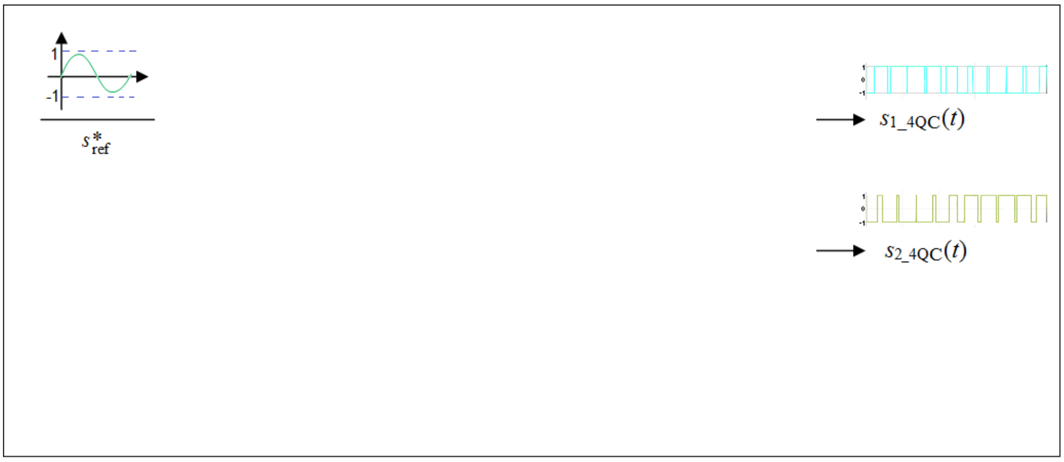


Figure 3.2: PWM unit for interleaved switching mode

3.2 Derive the formula for the current ripple Δi_2 in dependence of the DC-link voltage U_{DC}, the duty cycle s^* and the switching frequency f_S for both complementary and interleaved switching mode.

3.3 Determine the value of the secondary side leakage inductance $L_{\sigma s}$ so that the maximum occurring inductor current ripple $\Delta i_{2_{\max}}$ is 200 A for interleaved switching mode. How large is the maximum current ripple for complementary switching mode?

3.4 The output voltage of the 4QC is adjusted by a voltage controller. Furthermore the controller operates in power factor mode which means that the fundamental grid voltage and current are in phase so that only active power is drawn from the grid. Draw the phasor-diagrams qualitatively for the three output power cases: $P_{\text{out1}} = 0\,\text{MW}$, $P_{\text{out2}} = 0.5\,\text{MW}$ and $P_{\text{out3}} = 1.0\,\text{MW}$. Calculate the RMS-value I_{AC} of the input current i_{AC} and the phase angle φ between the grid voltage and the converter fundamental voltage for each of the three cases.

3.5 Determine the reactive power Q of the transformer for all three output powers listed in 3.4). Where does the reactive power, which is required by the transformer's leakage inductance, come from? Draw your calculated values in the diagram in Figure 3.3 and try to sketch the characteristic of Q over P. What is the mathematical relation between reactive and active power?

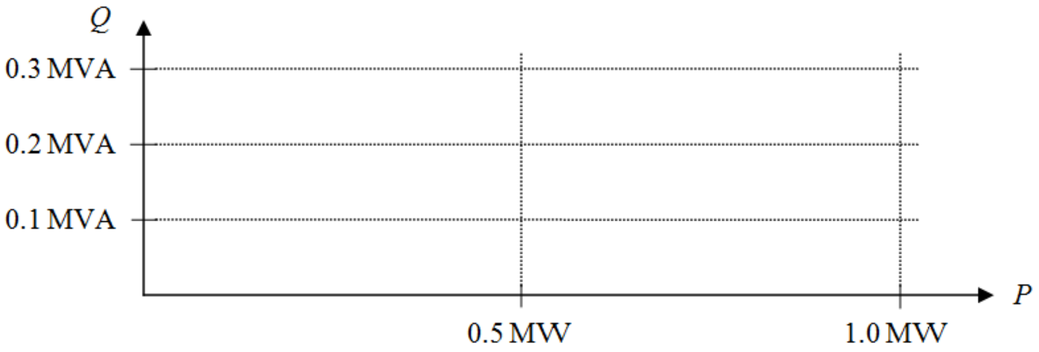


Figure 3.3: Diagram for reactive power over active power

3.6 The train is now decelerating. During this stage the controller receives a constant phase angle φ of -20° (see 3.4 for the definition of φ). Determine the active power P that is converted by the traction drives (generator mode) and fed into the grid. Determine the amount of recuperated energy W when the decelerating process lasts 30 seconds.

Task 4: Line-Commutated Rectifier (25 Credits)

A 3-pulse thyristor-controlled converter is connected to the 3-phase grid via a Dy-transformer configuration and supplies the excitation winding of a DC-motor. The following data are given for the assembly:

- Transformer secondary line-to-neutral voltage: $U_\mathrm{S} = 220\ \mathrm{V}$
- Rated current of excitation winding: $I_\mathrm{dN} = 5\ \mathrm{A}$
- Control angle at rated operation: $\alpha_\mathrm{N} = 60°$
- Overlapping/commutation angle at rated operation: $u_\mathrm{N} = 15°$

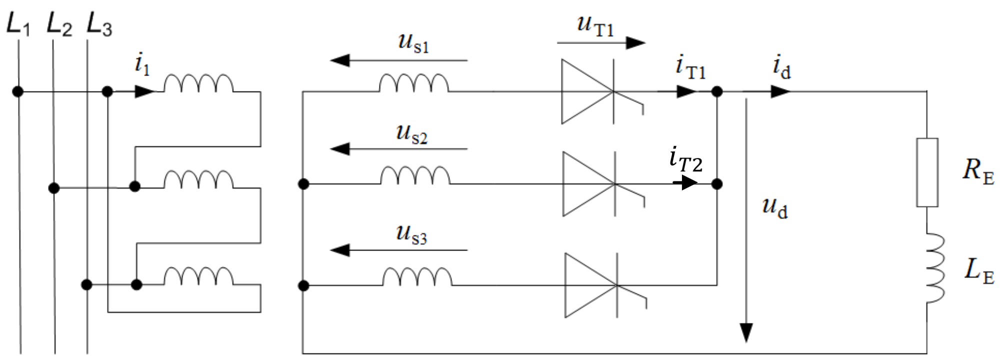
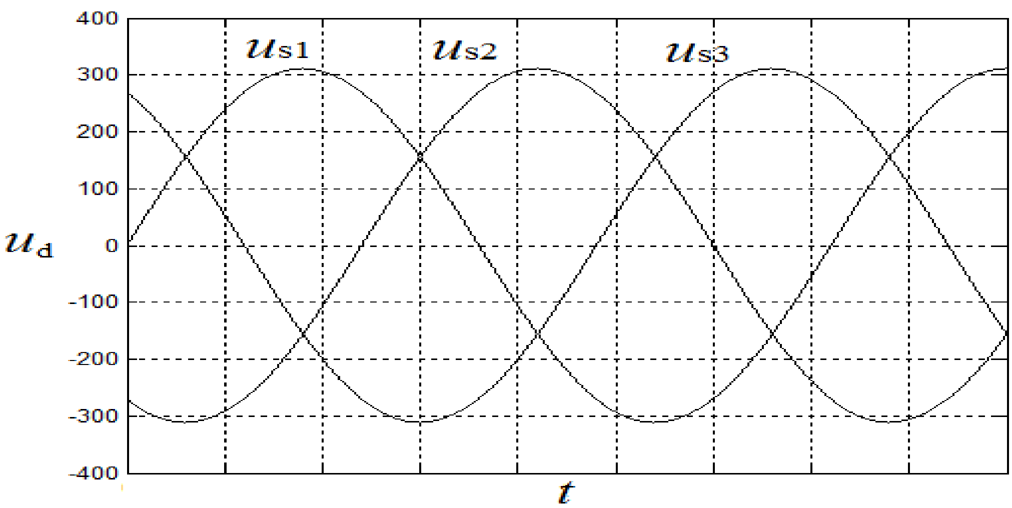


Figure 4.1: Line commutated rectifier in M3 configuration

Semiconductor and inductor losses can be neglected. Assume that the output current i_d is ideally smooth.

4.1 Assume steady state condition and sketch the voltage and current waveforms u_d, u_T1, i_T1, i_T2 and i_1 at rated operation in the diagrams in Figure 4.2.



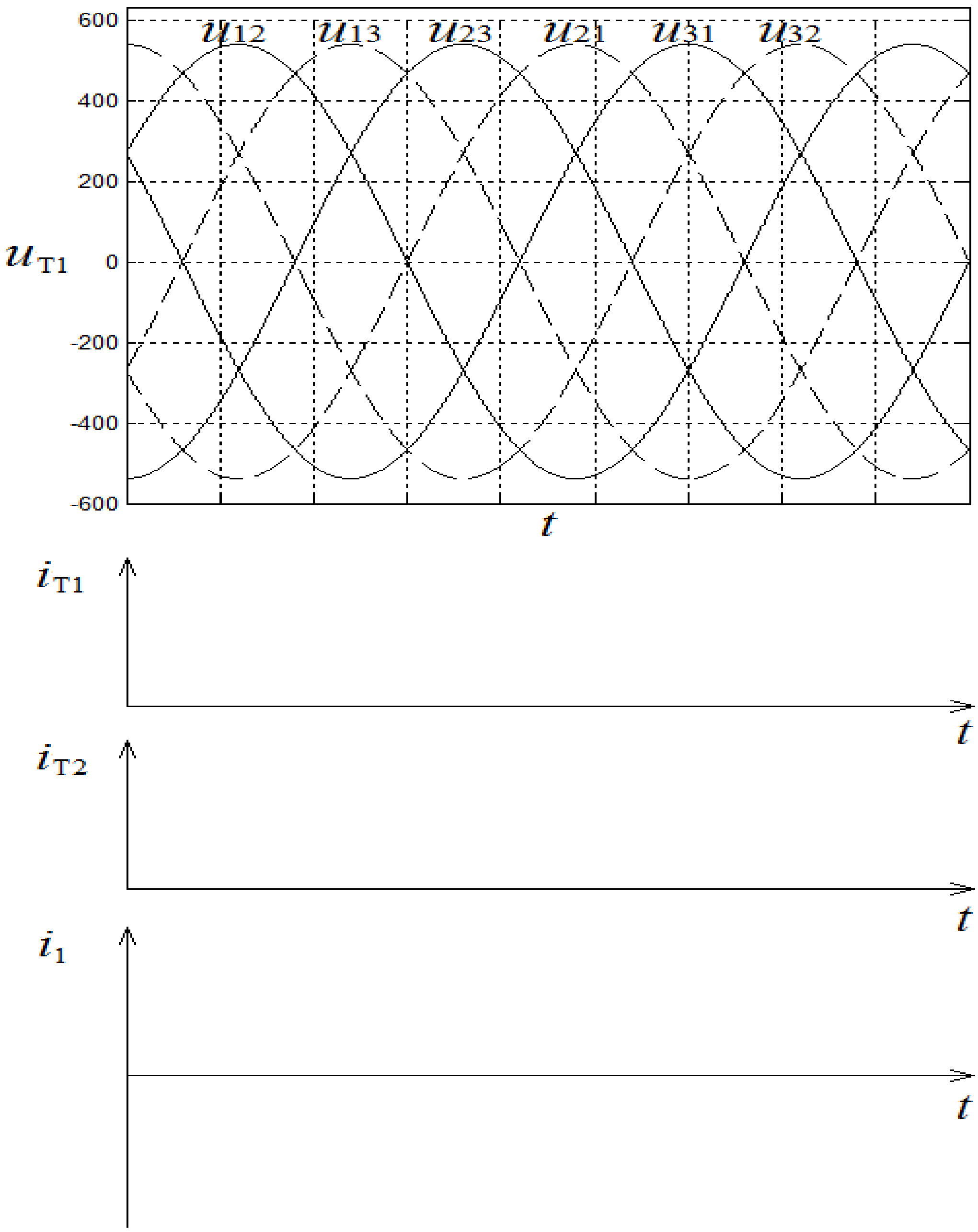


Figure 4.2: Diagrams for voltage and current waveforms

4.2 Draw the DC-equivalent circuit and calculate the effective resistance of the excitation circuit taking commutational voltage drops into account. The latter is calculated by $D_{\mathrm{x}} = p \cdot f \cdot L_{\mathrm{K}} \cdot I_{\mathrm{d}}$, where p is the pulse number and L_{K} is the commutation inductor (not shown in Figure 4.1; part of the transformer). The value of L_{K} can be derived from the relation $dx = \frac{D_{\mathrm{x}}}{U_{\mathrm{di}}} = \frac{1}{2}(\cos(\alpha_{\mathrm{N}}) - \cos(\alpha_{\mathrm{N}} + u_{\mathrm{N}}))$ with $U_{\mathrm{di}} = \frac{p}{\pi}\hat{u}_{\mathrm{d}} \sin\left(\frac{\pi}{p}\right)$.

4.3 Consider the start-up of the system. Calculate the minimum time t_3 required for building up the rated current $I_{d\mathrm{N}}$ in the fastest way. The total inductance value of the assembly on the excitation side is 18 H. The voltage ripple can be neglected.

For the remaining tasks a specification for the so far neglected current ripple of i_d should be used for calculating the excitation inductance approximately. Instead the AC-side inductance is neglected and the voltage drop across the resistance of the excitation R_E should be replaced by an equivalent voltage source $E = U_{\mathrm{di}\alpha}$, i. e. output voltage at $\alpha = 60°$. The converter can be thought of as supplying an active load with counter-voltage E and inductance L_E.

4.4 Sketch the output voltage u_d and current i_d waveforms for $\alpha_\mathrm{N} = 60°$ in the diagrams in Figure 4.3.

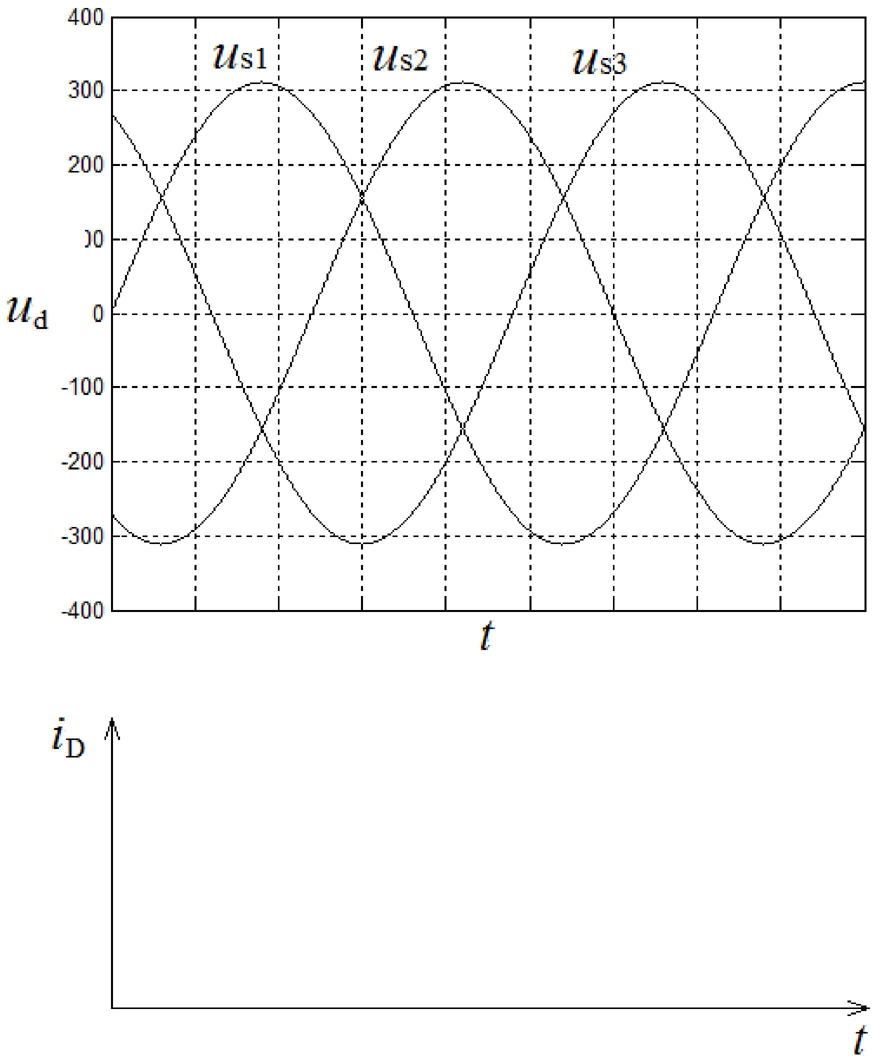


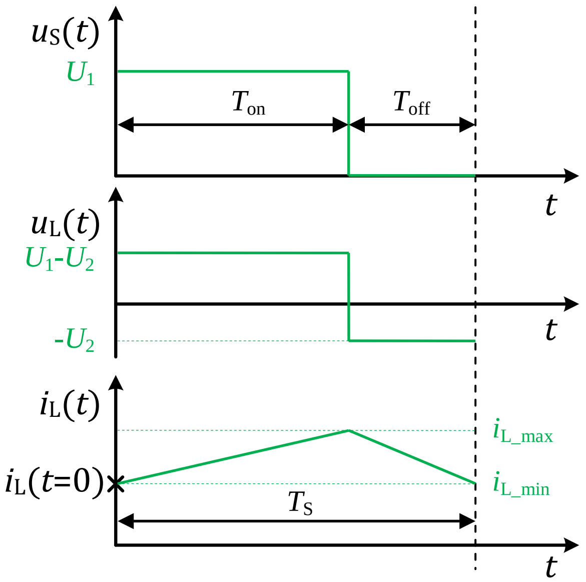
Figure 4.3: Diagrams for output voltage and output current

4.5 Calculate the inductance value of L_E using the aforementioned assumptions, if the current ripple Δi_d should be less than 1% or rated output current I_dN.

Solution

Task 1) Two-Quadrant Converter **[25 Credits]**

1.1) Waveform of switched voltage, inductor voltage and inductor current:



[5 Credits]

1.2) Duty cycle:

$$D = \frac{U_2}{U_1} = \frac{14\text{ V}}{48\text{ V}} = \underline{\underline{0.29}}$$

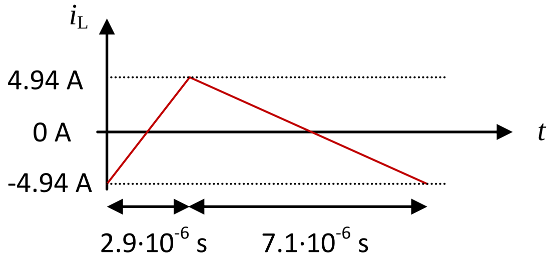
[2 Credits]

1.3) Inductor current ripple:

$$\Delta i_L = \frac{D(1-D)T_S U_1}{L} = \frac{0.29(1-0.29)\cdot 0.00001\text{ s}\cdot 48\text{ V}}{0.00001\text{ H}} = \underline{\underline{9.88\text{ A}}}$$

[2 Credits]

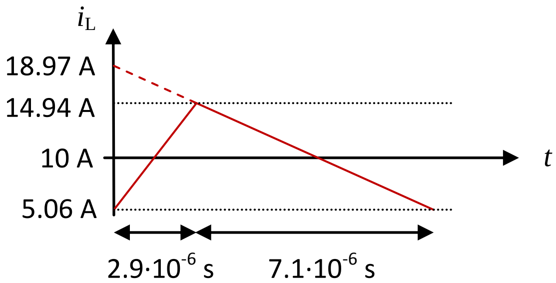
1.4) Inductor current RMS-value for zero output current:



$$I_L = \sqrt{I_2^2 + \Delta I_L^2} = \sqrt{I_2^2 + \left(\tfrac{1}{2\sqrt{3}}\Delta i_L\right)^2} = \sqrt{(0\text{ A})^2 + \left(\tfrac{1}{2\sqrt{3}}\cdot 9.88\text{ A}\right)^2} = \underline{\underline{2.85\text{ A}}}$$

[4 Credits]

1.5) Inductor current RMS-value for 10 A RMS output current:



$$I_L = \sqrt{I_2^2 + \Delta I_L^2} = \sqrt{I_2^2 + \left(\tfrac{1}{2\sqrt{3}}\Delta i_L\right)^2} = \sqrt{(10\text{ A})^2 + \left(\tfrac{1}{2\sqrt{3}}\cdot 9.88\text{ A}\right)^2} = \underline{\underline{10.399\text{ A}}}$$

Alternative approach:

$$I_L = \sqrt{\frac{1}{T_S}\left(\int_{0\text{ s}}^{2.9\cdot10^{-6}\text{s}}\left(5.06\text{ A} + \frac{9.88\text{ A}}{2.9\cdot10^{-6}\text{ s}}t\right)^2 dt + \int_{2.9\cdot10^{-6}\text{s}}^{10\cdot10^{-6}\text{s}}\left(18.97\text{ A} - \frac{9.88\text{ A}}{7.1\cdot10^{-6}\text{ s}}t\right)^2 dt\right)}$$

$$I_L = \sqrt{108.14\text{ A}^2} = \underline{\underline{10.4\text{ A}}}$$

[5 Credits]

1.6) Losses of converter:

$$P_V|_{I_2=0\text{ A}} = R_L I_L^2 = 0.01\,\Omega\cdot(2.85\text{ A})^2 = \underline{\underline{0.081\text{ W}}}$$

$$P_V|_{I_2=10\text{ A}} = R_L I_L^2 = 0.01\,\Omega\cdot(10.4\text{ A})^2 = \underline{\underline{1.082\text{ W}}}$$

[4 Credits]

1.7) Efficiency of converter:

$$\eta|_{I_2=0\text{ A}} = \frac{P_\text{out}}{P_\text{in}} = \frac{P_\text{out}}{P_\text{out}+P_\text{V}} = \frac{48\text{ V}\cdot 0\text{ A}}{48\text{ V}\cdot 0\text{ A}+0.081\text{ W}} = \underline{\underline{0}} \quad (\triangleq 0\%)$$

$$\eta|_{I_2=10\text{ A}} = \frac{P_\text{out}}{P_\text{in}} = \frac{P_\text{out}}{P_\text{out}+P_\text{V}} = \frac{48\text{ V}\cdot 10\text{ A}}{48\text{ V}\cdot 10\text{ A}+1.082\text{ W}} = \underline{\underline{0.9977}} \quad (\triangleq 99.77\%)$$

[3 Credits]

Task 2) Boost Converter **[25 Credits]**

2.1) Output current and load resistor:

$$I_2 = \frac{P_2}{U_2} = \frac{1800\ \mathrm{W}}{400\ \mathrm{V}} = \underline{\underline{4.5\ \mathrm{A}}}$$

$$R_\mathrm{L} = \frac{U_2}{I_2} = \frac{400\ \mathrm{V}}{4.5\ \mathrm{A}} = \underline{\underline{88.89\ \Omega}}$$

[2 Credits]

2.2) Duty cycle and input voltage:

$$U_1 = \frac{P_2}{I_\mathrm{L}} = \frac{1800\ \mathrm{W}}{15\ \mathrm{A}} = \underline{\underline{120\ \mathrm{V}}}$$

$$D = \frac{U_2 - U_1}{U_2} = \frac{400\ \mathrm{V} - 120\ \mathrm{V}}{400\ \mathrm{V}} = \underline{\underline{0.7}}$$

Alternative approach:

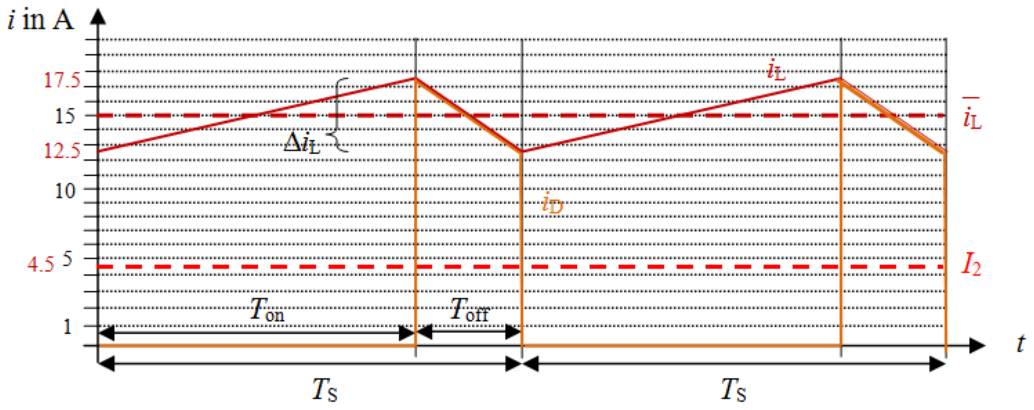
$$D = \frac{I_\mathrm{L} - I_2}{I_\mathrm{L}} = \frac{15\ \mathrm{A} - 4.5\ \mathrm{A}}{15.\mathrm{A}} = \underline{\underline{0.7}}$$

$$U_1 = U_2(1 - D) = 400\ \mathrm{V}(1 - 0.7) = \underline{\underline{120\ \mathrm{V}}}$$

[3 Credits]

2.3) Inductor current ripple and diagrams:

$$\Delta i_\mathrm{L} = \frac{D T_\mathrm{S} U_1}{L} = \frac{0.7 \cdot 0.0001\ \mathrm{s} \cdot 120\ \mathrm{V}}{0.00168\ \mathrm{H}} = \underline{\underline{5\ \mathrm{A}}}$$



[7 Credits]

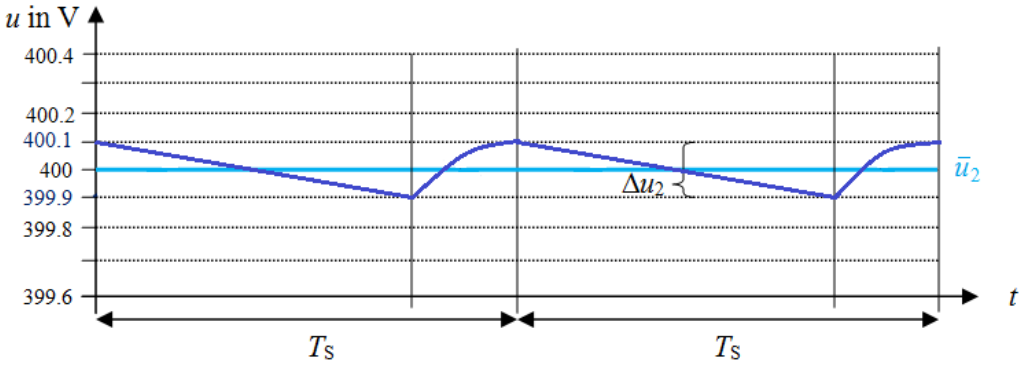
2.4) Load resistance value for boundary conduction mode:

$$\overline{i_{\mathrm{L}}} = \frac{\Delta i_{\mathrm{L}}}{2} = \frac{I_2}{1-D} = \frac{U_2}{(1-D)R'_{\mathrm{L}}} \rightarrow R'_{\mathrm{L}} = \frac{2U_2}{(1-D)\Delta i_{\mathrm{L}}} = \frac{2\cdot 400\ \mathrm{V}}{0.3\cdot 5\ \mathrm{A}} = \underline{\underline{533{,}33\ \Omega}}$$

[4 Credits]

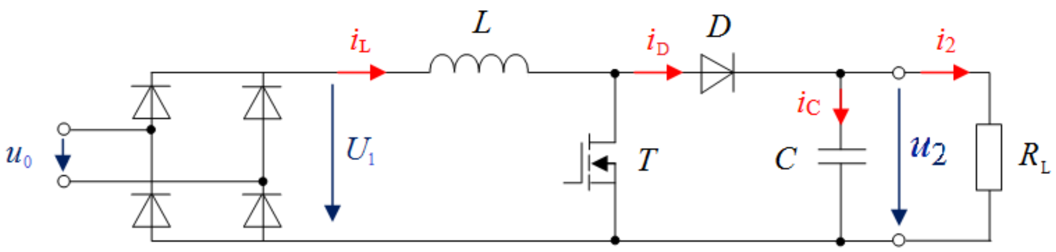
2.5) Capacitor value:

$$\Delta u_2 = \frac{I_2 D T_{\mathrm{S}}}{C} \rightarrow C = \frac{I_2 D T_{\mathrm{S}}}{\Delta u_2} = \frac{4.5\ \mathrm{A}\cdot 0.7\cdot 0.0001\ \mathrm{s}}{0.2\ \mathrm{V}} = \underline{\underline{1.575\ \mathrm{mF}}}$$



[6 Credits]

2.6) Extension to PFC-rectifier:

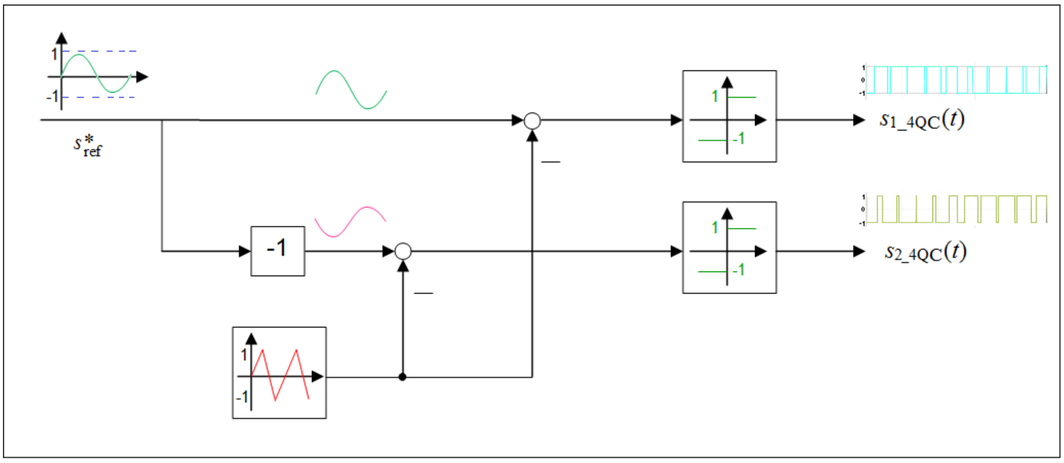


A full bridge consisting of four diodes has to be inserted between the AC-voltage source and the boost converter.

[3 Credits]

Task 3) Four-Quadrant Converter **[25 Credits]**

3.1) PWM unit structure for interleaved operation mode of 4QC:



[4 Credits]

3.2) Derivation of formula for inductor current ripple:

$$\Delta i_{2\_\text{INTERLEAVED}} = |s^*|(1-|s^*|)\cdot\frac{T_S U_1}{2L} = |s^*|(1-|s^*|)\cdot\frac{U_{\text{DC}}}{2f_S\cdot L_{\sigma s}}$$

- Voltage swing (Spannungshub) is U_{DC}
- Maximum ripple for s^* = +/- 0.5

$$\Delta i_{2\_\text{COMPLEMENTARY}} = (1-s^{*2})\cdot\frac{T_S U_1}{2L} = (1-s^{*2})\frac{U_{\text{DC}}}{2f_S\cdot L_{\sigma s}}$$

- Voltage swing (Spannungshub) is U_{DC}
- Maximum ripple for s^* = 0

[4 Credits]

3.3) Leakage inductance for requested maximum current ripple:

$$\Delta i_{2\_\text{INTERLEAVED}} = |s^*|(1-|s^*|)\cdot\frac{U_{\text{DC}}}{2f_S\cdot L_{\sigma s}} \rightarrow L_{\sigma s} = \frac{U_{\text{DC}}\cdot 0.25}{\Delta i_{2\_\text{INTERLEAVED}}\cdot 2f_S}$$

$$L_{\sigma s} = \frac{2000\ \text{V}\cdot 0.25}{200\ \text{A}\cdot 2\cdot 300\ \text{Hz}} = \underline{\underline{4.167\ \text{mH}}}$$ (s^* = +/- 0.5 leads to maximum ripple)

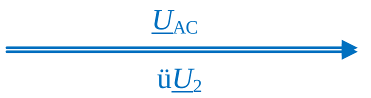
$$\Delta i_{2\_\text{COMPLEMENTARY\_max}} = (1 - s^{*2}) \frac{U_{\text{DC}}}{2 f_S \cdot L_{\sigma s}} = \frac{2000\ \text{V} \cdot 1}{300\ \text{Hz} \cdot 2 \cdot 0.004167\ \text{H}} = \underline{\underline{800\ \text{A}}}$$

[3 Credits] (s^* = 0 leads to maximum ripple)

3.4) Sketch Phasor diagrams for three output powers:

Case 1: P_{out} = 0 MW

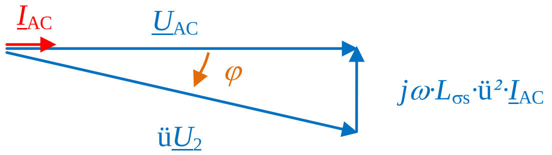
$$P = U_{\text{AC}} \cdot I_{\text{AC}} \cdot \cos(\varphi_{\text{ui}}) \rightarrow I_{\text{AC}} = \frac{0\ \text{MW}}{15\ \text{kV} \cdot \cos(0°)} = \underline{\underline{0\ A}}$$



$$\varphi = \arctan\left(\frac{\omega \cdot L_{\sigma s} \cdot \text{ü}^2 \cdot I_{\text{AC}}}{U_{\text{AC}}}\right) = \arctan\left(\frac{2\pi \cdot 16.667\ \text{Hz} \cdot 0.004167\ \text{H} \cdot 12^2 \cdot 0\ \text{A}}{15\ \text{kV}}\right) = \underline{\underline{0°}}$$

Case 2: P_{out} = 0.5 MW

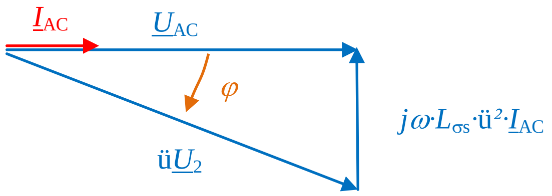
$$P = U_{\text{AC}} \cdot I_{\text{AC}} \cdot \cos(\varphi_{\text{ui}}) \rightarrow I_{\text{AC}} = \frac{0.5\ \text{MW}}{15\ \text{kV} \cdot \cos(0°)} = \underline{\underline{33.33\ A}}$$



$$\varphi = \arctan\left(\frac{\omega \cdot L_{\sigma s} \cdot \text{ü}^2 \cdot I_{\text{AC}}}{U_{\text{AC}}}\right) = \arctan\left(\frac{2\pi \cdot 16.667\ \text{Hz} \cdot 0.004167\ \text{H} \cdot 12^2 \cdot 33.33\ \text{A}}{15\ \text{kV}}\right) = \underline{\underline{7.948°}}$$

Case 3: P_{out} = 1.0 MW

$$P = U_{\text{AC}} \cdot I_{\text{AC}} \cdot \cos(\varphi_{\text{ui}}) \rightarrow I_{\text{AC}} = \frac{1.0\ \text{MW}}{15\ \text{kV} \cdot \cos(0°)} = \underline{\underline{66.66\ A}}$$



$$\varphi = \arctan\left(\frac{\omega \cdot L_{\sigma s} \cdot \text{ü}^2 \cdot I_{\text{AC}}}{U_{\text{AC}}}\right) = \arctan\left(\frac{2\pi \cdot 16.667\ \text{Hz} \cdot 0.004167\ \text{H} \cdot 12^2 \cdot 66.66\ \text{A}}{15\ \text{kV}}\right) = \underline{\underline{15.6°}}$$

[5 Credits]

3.5) Reactive Power:

Case 1: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_1 \cdot ü^2)^2 = 2\pi \cdot 16.667\ \mathrm{Hz} \cdot 0.004167\ \mathrm{H} \cdot (0\ \mathrm{A} \cdot 144)^2$

$Q = \underline{\underline{0\ \mathrm{MVA}}}$

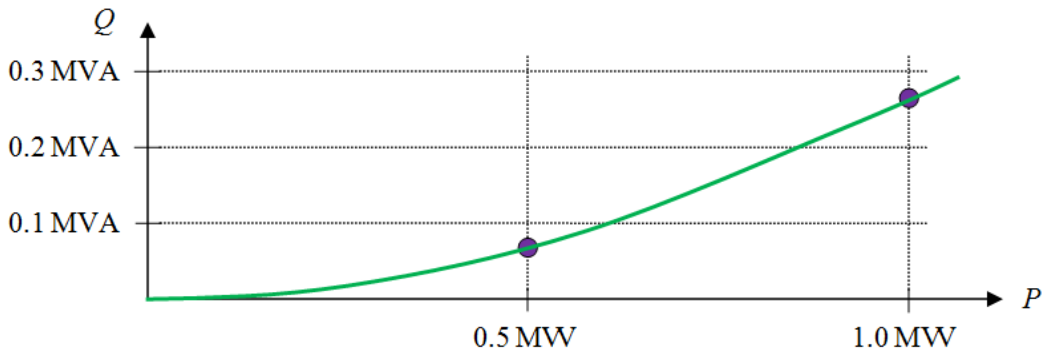
Case 2: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_1 \cdot ü^2)^2 = 2\pi \cdot 16.667\ \mathrm{Hz} \cdot 0.004167\ \mathrm{H} \cdot (33.33\ \mathrm{A} \cdot 144)^2$

$Q = \underline{\underline{0.0698\ \mathrm{MVA}}}$

Case 3: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_1 \cdot ü^2)^2 = 2\pi \cdot 16.667\ \mathrm{Hz} \cdot 0.004167\ \mathrm{H} \cdot (66.66\ \mathrm{A} \cdot 144)^2$

$Q = \underline{\underline{0.2793\ \mathrm{MVA}}}$

Reactive power has to be provided by the 4QC. The reactive power oscillates between transformer and 4QC.



Quadratic relation between required reactive power by transformer and active power drawn from the grid to supply the load.

[5 Credits]

3.6) Recuperated active power:

$$\tan(\varphi) = \frac{2\pi \cdot f \cdot L_{\sigma s} \cdot ü^2 \cdot I_{AC} \cdot U_{AC}}{U_{AC} \cdot U_{AC}} = \frac{2\pi \cdot f \cdot L_{\sigma s} \cdot ü^2 \cdot P}{U_{AC}^2} \rightarrow P = \frac{U_{AC}^2 \cdot \tan(\varphi)}{2\pi \cdot f \cdot L_{\sigma s} \cdot ü^2}$$

$$P = \frac{U_{AC}^2 \cdot \tan(\varphi)}{2\pi \cdot f \cdot L_{\sigma s} \cdot ü^2} = \frac{(15\ \mathrm{kV})^2 \cdot \tan(-20°)}{2\pi \cdot 16.667\ \mathrm{Hz} \cdot 0.004167\ \mathrm{H} \cdot 144} = \underline{\underline{-1.303\ \mathrm{MW}}}$$

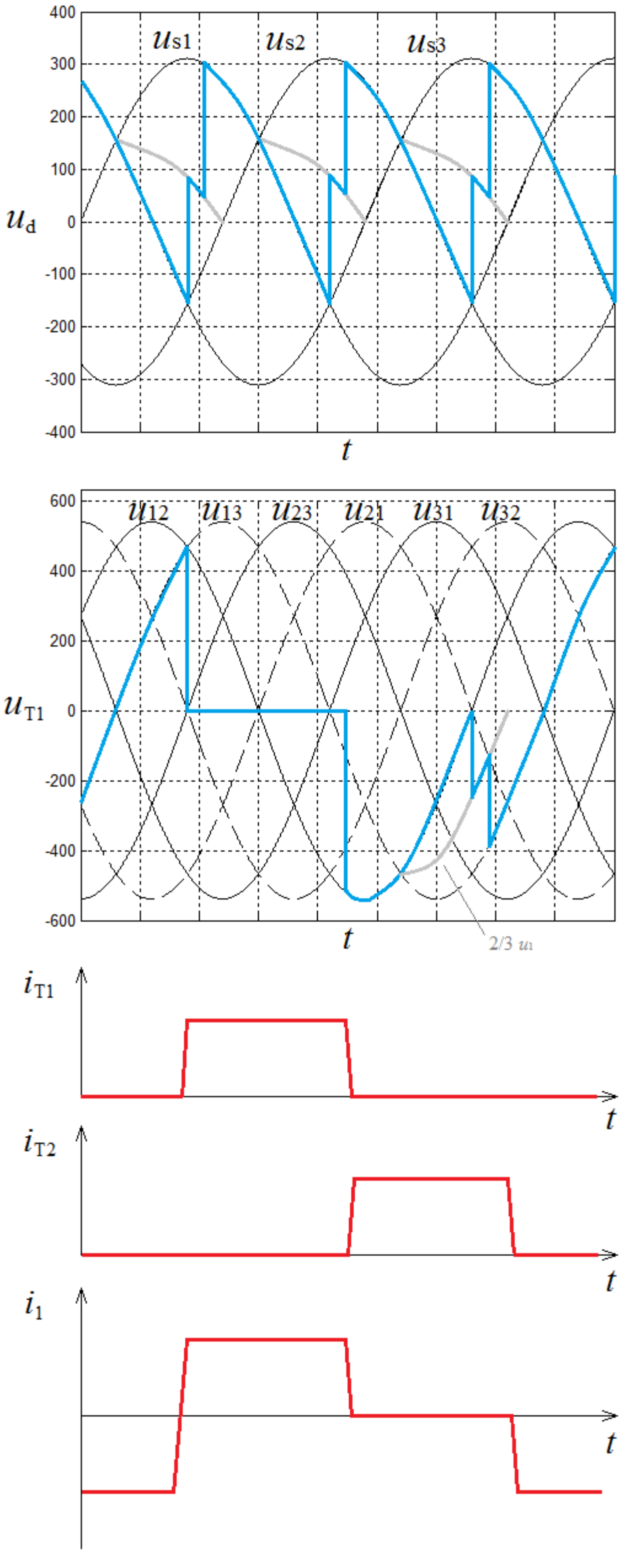
$$W = P \cdot \Delta t = 1.303\ \mathrm{MW} \cdot \frac{0.5\ \mathrm{min}}{60\ \mathrm{min}} \cdot 1\ \mathrm{h} = \underline{\underline{10.86\ \mathrm{kWh}}}$$

[4 Credits]

Task 4) Line-Commutated Rectifier **[25 Credits]**

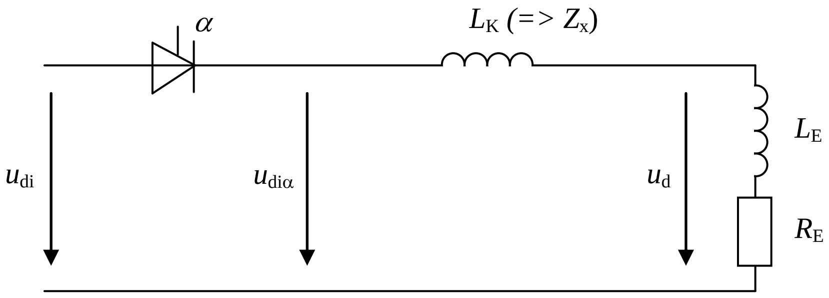
4.1) Voltage and current waveforms:

$\alpha_{\mathrm{N}} = 60°$ $\qquad u_{\mathrm{N}} = 15°$



[7 Credits]

4.2) Effective resistance of excitation circuit:



$$R_E = \frac{U_d}{I_d} = \frac{U_{di\alpha} - Z_x I_d}{I_d} = \frac{U_{di}}{I_{dn}} \cos(\alpha_n) - Z_x$$

$$U_{di} = \frac{p}{\pi} u_d \cdot \sin\left(\frac{\pi}{p}\right) = \frac{3}{\pi}\sqrt{2} \cdot 200\ \mathrm{V} \cdot \sin\left(\frac{\pi}{p}\right) = 257\ \mathrm{V}$$

$$Z_x = \frac{D_{xN}}{I_{dN}} = d_x \frac{U_{di}}{I_{dN}}$$

$$\cos(\alpha + u) \cos(\alpha) - 2d_x$$

$$d_x = \frac{1}{2}[\cos(\alpha_N) - \cos(\alpha_N + u_N)] = 0.12$$

$$Z_x = 0.12 \frac{257\ \mathrm{V}}{5\ \mathrm{A}} = 6.2\ \Omega$$

$$R_E = \frac{257\ \mathrm{V}}{5\ \mathrm{A}} \cos(60°) - 6.2\ \Omega = \underline{\underline{19.5\ \Omega}}$$

[5 Credits]

4.3) Required time to build up rated current:

$$I_d(t) = I_{d\infty} + (I_{d0} - i_{d\infty}) e^{-t/T}$$

with $T = \frac{L}{R_E + Z_x} = 700\ \mathrm{ms}$

and $I_{d\infty} = \frac{U_{di}}{R_E + Z_x}; I_{d0} = 0\ A$

For $t = t_3$:

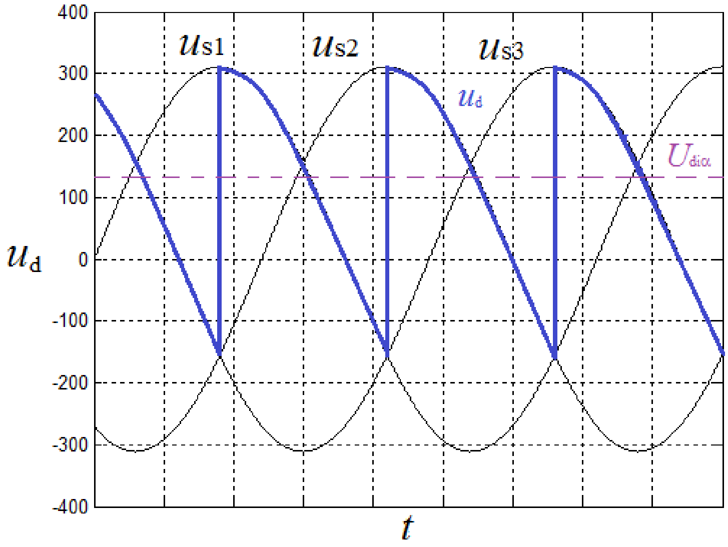
$$I_{dN} = I_{d\infty} - I_{d\infty} e^{-t_3/T}$$

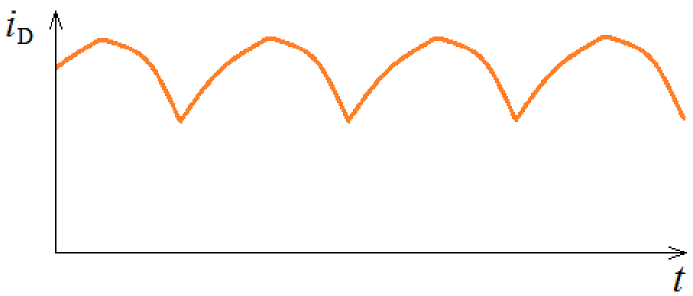
➔ $t_3 = T \cdot \ln\left(\frac{1}{1 - I_{dN}/I_{d\infty}}\right) = T \cdot \ln\left(\frac{1}{1 - \frac{(R_E + Z_x) I_{dN}}{U_{di}}}\right) = \underline{\underline{485\ \mathrm{ms}}}$

[**4 Credits]**

4.4) Output voltage and current waveforms:

$\alpha_{\mathrm{N}} = 60°$ $\quad u_{\mathrm{N}} = 0°$





[2 Credits]

4.5) Determination of inductance value:

$$\frac{di_{\mathrm{d}}}{dt} = \frac{1}{L_{\mathrm{E}}}\int_0^t (u_{\mathrm{d}}(t) - R_{\mathrm{E}} i_{\mathrm{d}})dt + i_{\mathrm{d}}(0) \approx \frac{1}{L_{\mathrm{E}}}\int_0^t (u_{\mathrm{d}}(t) - U_{\mathrm{di}\alpha})dt + i_{\mathrm{d}}(0)$$

$$\frac{di_{\mathrm{d}}}{dt} \geq 0 \; für \; 0 \leq t \leq t_1$$

$$u_{\mathrm{d}}(t_1) - U_{\mathrm{di}\alpha} = 0$$

$$\hat{u}_{\mathrm{s}} \cdot \cos(\omega t_1) - \frac{p}{\pi}\hat{u}_{\mathrm{s}} \cdot \sin\left(\frac{\pi}{p}\right) \cdot \cos(\alpha) = 0$$

$$\cos(\omega t_1) - \frac{p}{\pi}\sin\left(\frac{\pi}{p}\right) \cdot \cos(\alpha) = 0$$

$$\omega t_1 = \arccos\left(\frac{p}{\pi}\sin\left(\frac{\pi}{p}\right) \cdot \cos(\alpha)\right) = 65.6°$$

$$\Delta i_{\mathrm{d}} = i_{\mathrm{d}}(t_1) - i_{\mathrm{d}}(0) = \frac{1}{\omega L_{\mathrm{E}}}\int_0^{\omega t_1} (\hat{u}_{\mathrm{s}} \cdot \cos(\omega t) - U_{\mathrm{di}\alpha})d\omega t$$

$$\Delta i_{\mathrm{d}} = \frac{1}{\omega L_{\mathrm{E}}}\left[\sqrt{2}U_{\mathrm{s}} \cdot \sin(\omega t_1) - U_{\mathrm{di\alpha}}\omega t_1\right]$$

$$L_{\mathrm{E}} = \frac{1}{\omega \cdot 0.01 I_{\mathrm{dN}}}\left[\sqrt{2}U_{\mathrm{s}} \cdot \sin(\omega t_1) - U_{\mathrm{di}}\cos(\alpha) \cdot \omega t_1\right]$$

$$L_{\mathrm{E}} = \frac{\sqrt{2}U_{\mathrm{s}}}{\omega \cdot 0.01 I_{\mathrm{dN}}}\left[\sin(\omega t_1) - \frac{p}{\pi}\sin\left(\frac{\pi}{p}\right) \cdot \cos(\alpha) \cdot \frac{\pi \cdot 65.6°}{180°}\right] = \underline{\underline{8.66\ \mathrm{H}}}$$

[7 Credits]