Department of Power Electronics and Electrical Drives

Prof. Dr. Ing. Joachim Böcker

Power Electronics

08.09.2014

Surname:	**Student number:**
First name:	
Course of study:	

Task: (Points)	1 (25)	2 (25)	3 (25)	4 (25)		Total (100)		Mark

Duration: 120 minutes

Permitted resources:

- a non-programmable calculator without graphic display
- drawing materials (compasses, protractor, ruler, pens ...)

Please note:

- Please prepare your student ID card (with photo) on your desk for the attendance check.
- Please label each exam sheet with your name and student number. Use a new exam sheet for each task. Do not use pencils or red pens.
- With numerical calculations, the units must be considered in every step. Not following that rule will result in deduction of points.
- All solutions must be clearly documented and wherever required explained! The entry of a mere final result without any approach will not be counted.

Good Luck!

Task 1: Boost Converter (25 Points)

Figure 1 shows a boost converter. The input voltage to the converter is $U_1 = 8\,\mathrm{V}$. The average output voltage $\bar{u}_2$ should be $32\,\mathrm{V}$ while delivering a load with a DC content of $4\,\mathrm{A}$. The power-switches are assumed to be ideal during conduction and blocking states (0 V during conduction and 0 A in blocking). The converter is operated at a switching frequency of 25 kHz. The equivalent series resistance of the capacitor is given as $R_C = 25\,\mathrm{m\Omega}$ and the capacitance as $C = 500\,\mathrm{\mu F}$.

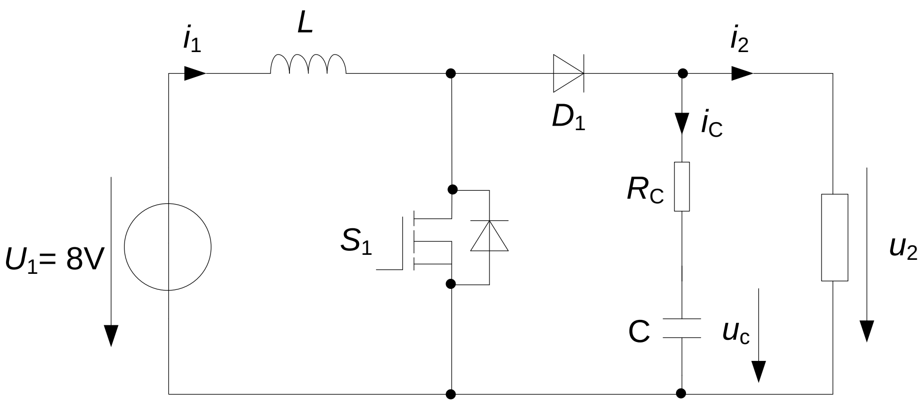


Figure 1: Boost converter

Note: For questions 1.1 to 1.5, R_C can be considered zero.

Estimate the following:

1.1 The DC content of the input current i_1.

$$\frac{U_2}{U_1} = \frac{1}{1-D}$$

$$D = 1 - \frac{U_1}{U_2} = 1 - \frac{8\,\mathrm{V}}{32\,\mathrm{V}} = 1 - \frac{1}{4} = 0.75$$

$$U_1 \cdot I_1 = U_2 \cdot I_2$$

$$I_1 = \frac{U_2 I_2}{U_1} = \frac{32\,\mathrm{V} \cdot 4\,\mathrm{A}}{8\,\mathrm{V}} = 16\,\mathrm{A}$$

1.2 Evaluate the inductance L to limit the ripple current of i_1 to 20% of the DC content (assume the output voltage approximately constant).

$$U_1 = L\,\frac{di}{dt}$$

$$U_1 = L\,\frac{\Delta i}{\Delta t}$$

$$L = \frac{U_1\,\Delta t}{\Delta i}$$

$$\Delta t = DT_S = \frac{0.75}{25\cdot 10^3\,\mathrm{s}^{-1}}$$

$$\Delta i = 20\,\%\;\; i_1 = 0.2\cdot 16\,\mathrm{A} = 3.2\,\mathrm{A}$$

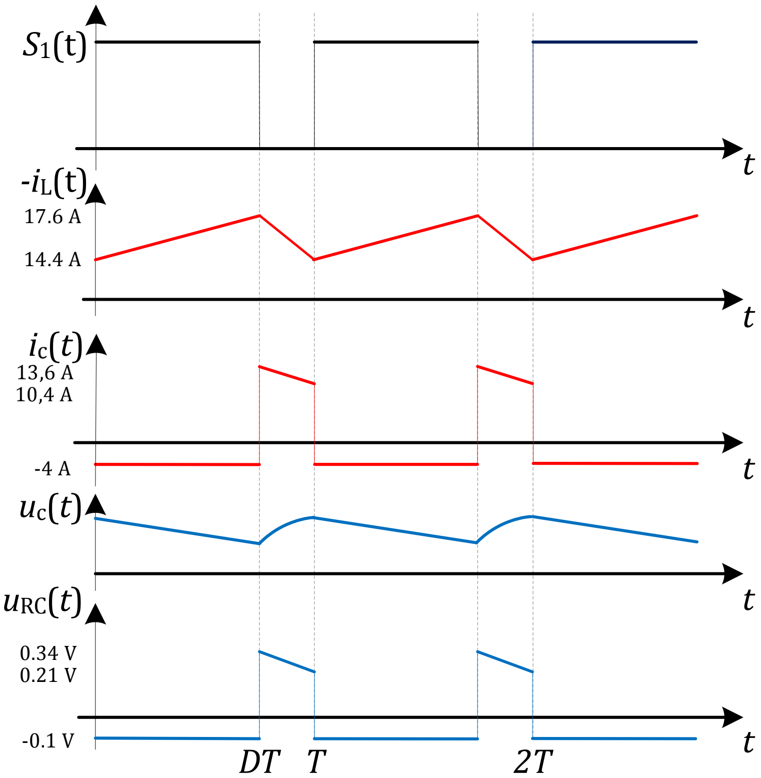
$$L = 8\,\mathrm{V}\cdot\frac{0.75\,\mathrm{s}}{25\cdot 10^3}\cdot\frac{1}{3.2\,\mathrm{A}} = 75\,\mu\mathrm{H}$$

1.3 For what value of the inductance L does the converter operate at the boundary between continuous and discontinuous conduction modes?

At the boundary, $\Delta i = 2\cdot I_1 = 2\cdot 16\,\mathrm{A} = 32\,\mathrm{A}$

$$L = U_1\,\frac{\Delta t}{\Delta i} = 8\,\mathrm{V}\,\frac{0.75\,\mathrm{s}}{25\cdot 10^3}\frac{1}{32\,\mathrm{A}} = 7.5\,\mu\mathrm{H}$$

Sketch the current through the capacitor C (assume the load current approximately constant)



1.4 Sketch u_C.

1.5 Assume that the above results remain valid even for a small value of $R_C = 25\,\mathrm{m}\Omega$. Evaluate the ripple voltage across R_C.

Task 2: Buck converter (25 Points)

Figure 2 shows a non-ideal buck converter. The source and inductor resistances are represented by R_1 and R_L respectively. The devices S_1 and D_1 are assumed to be ideal. The state variables are the inductor current i_L and capacitor voltage u_2. The circuit is to be operated in continuous conduction mode.

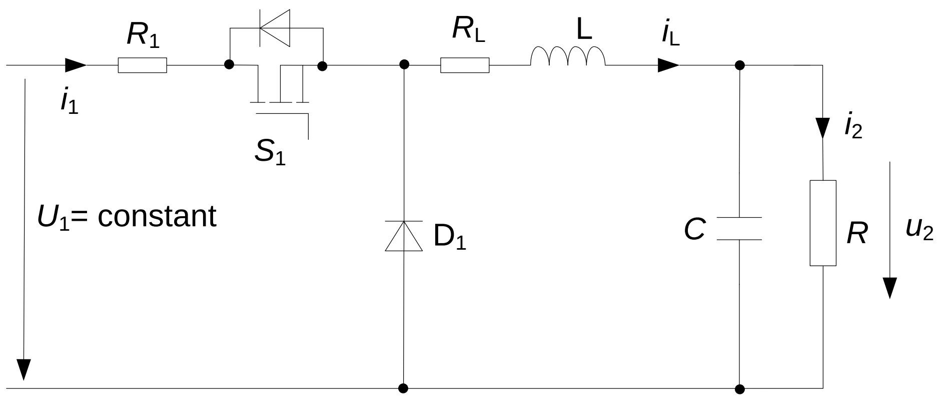


Figure 2: Buck converter

2.1 Write down the differential equations of the converter for the two state variables during ON state i.e. when S_1 is ON.

$$U_1 = (R_1 + R_L)i_L + L\frac{di_L}{dt} + u_2$$

$$L\frac{di_L}{dt} = U_1 - (R_1 + R_L)i_L - u_2$$

$$\frac{di_L}{dt} = \frac{U_1}{L} - \frac{(R_1 + R_2)}{L}i_L - \frac{u_2}{L}$$

$$i_L = C\frac{du_2}{dt} + \frac{u_2}{R}$$

$$\frac{du_2}{dt} = \frac{i_L}{C} - \frac{u_2}{RC}$$

2.2 Write down the differential equations of the converter for the two state variables during OFF state i.e. when S_1 is OFF.

$$L\frac{di_L}{dt} = -u_2 - R_L i_L$$

$$\frac{di_L}{dt} = \frac{-u_2}{L} - \frac{R_L i_L}{L}$$

$$i_L = C\,\frac{du_2}{dt} + \frac{u_2}{R}$$

$$\frac{du_2}{dt} = \frac{i_L}{C} - \frac{u_2}{R_c}$$

2.3 Using the above write down the averaged dynamic model of the converter in state space matrix notation.

On state: $\begin{bmatrix} \frac{di_L}{dt} \\ \frac{du_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-(R_1+R_L)}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} U_1$

Off state: $\begin{bmatrix} \frac{di_L}{dt} \\ \frac{du_2}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} U_1$

$$A_1 = \begin{bmatrix} \frac{-(R_1+R_2)}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{Rc} \end{bmatrix} \qquad b_1 = \begin{bmatrix} \frac{i}{L} \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \frac{-R_L}{C} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \qquad b_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A = A_1 d + A_2(1-d) \qquad b = b_1 d + b_2(1-d)$$

$$A = \begin{bmatrix} \frac{-R_1}{L} d - \frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \qquad b = \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{du_2}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_i}{L} d - \frac{R_L}{L} & -\frac{1}{L} \\ \frac{1}{c} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} d \\ 0 \end{bmatrix} U_1$$

2.4 Derive the gain $\bar{u}_2/U_1$ at steady state as a function of the duty cycle.

At steady state: $\frac{di_L}{dt} = 0 \quad \rightarrow \quad 0 = \left(-\frac{R_1}{L}\,d \;-\frac{R_L}{L}\right) i_L - \frac{u_2}{L} + \frac{U_1 d}{L}$

$$(R_1 d + R_L)\, i_L + u_2 = U_1 d$$

$$\frac{du_2}{dt} = 0 \quad \rightarrow \quad \frac{i_2}{C} - \frac{U_2}{RC} = 0 \quad \rightarrow \quad i_L = \frac{U_2}{R}$$

$$\left(\frac{R_1 d}{R} + \frac{R_L}{R} + 1\right) U_2 = U_1 d$$

$$\frac{U_2}{U_1} = \frac{d}{1 + R_L + \frac{R_i d}{R}}$$

$$\frac{I_2}{I_1} = \frac{1}{d}$$

2.5 Evaluate the efficiency of the converter based on the average modeling.

$$\eta = \frac{U_2\, I_2}{U_1\, I_1} = \frac{1}{1 + \frac{R_L}{R} + \frac{R_1 d}{R}}$$

Task 3: Four-Quadrant Converter (25 Points)

A four-quadrant converter (4QC) is connected to the single-phase grid. The overhead line delivers a single-phase sinusoidal AC voltage V_{AC} with grid frequency f. The two four-quadrant converters are operated in interleaving operation to minimize the current ripple. The traction transformer has two primary windings connected in parallel and two secondary windings.

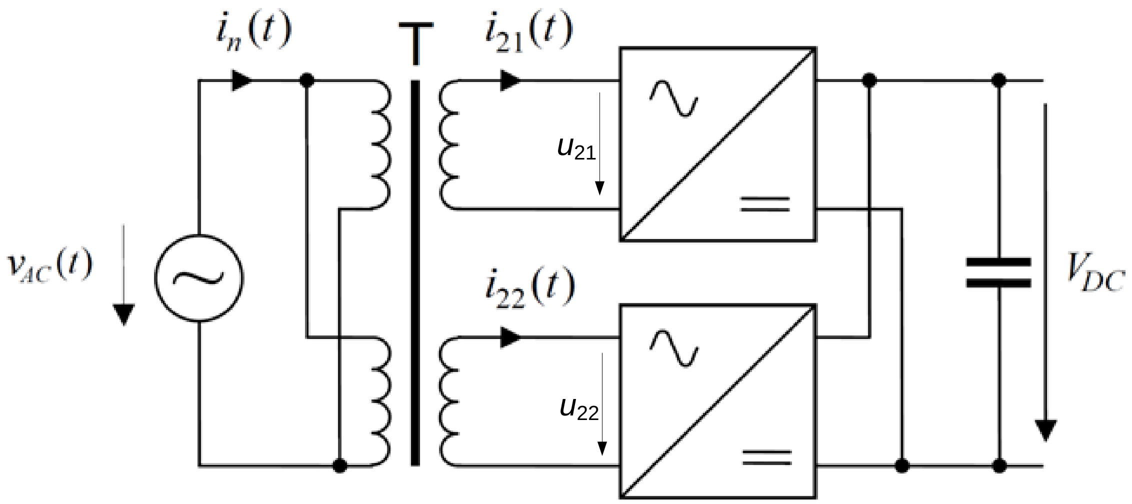
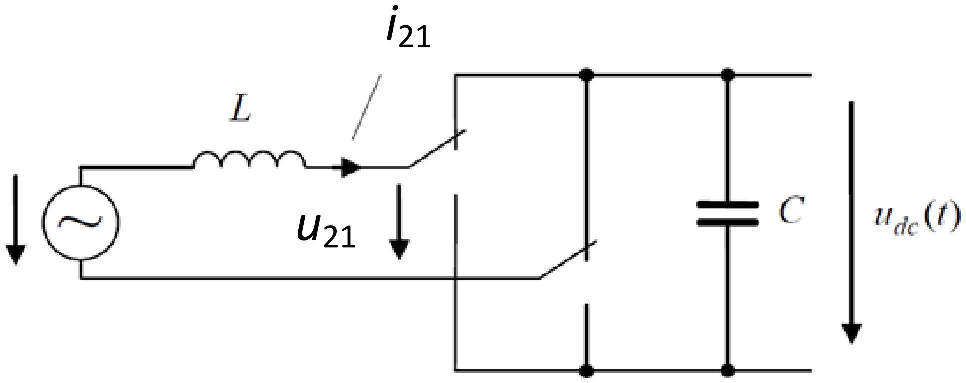


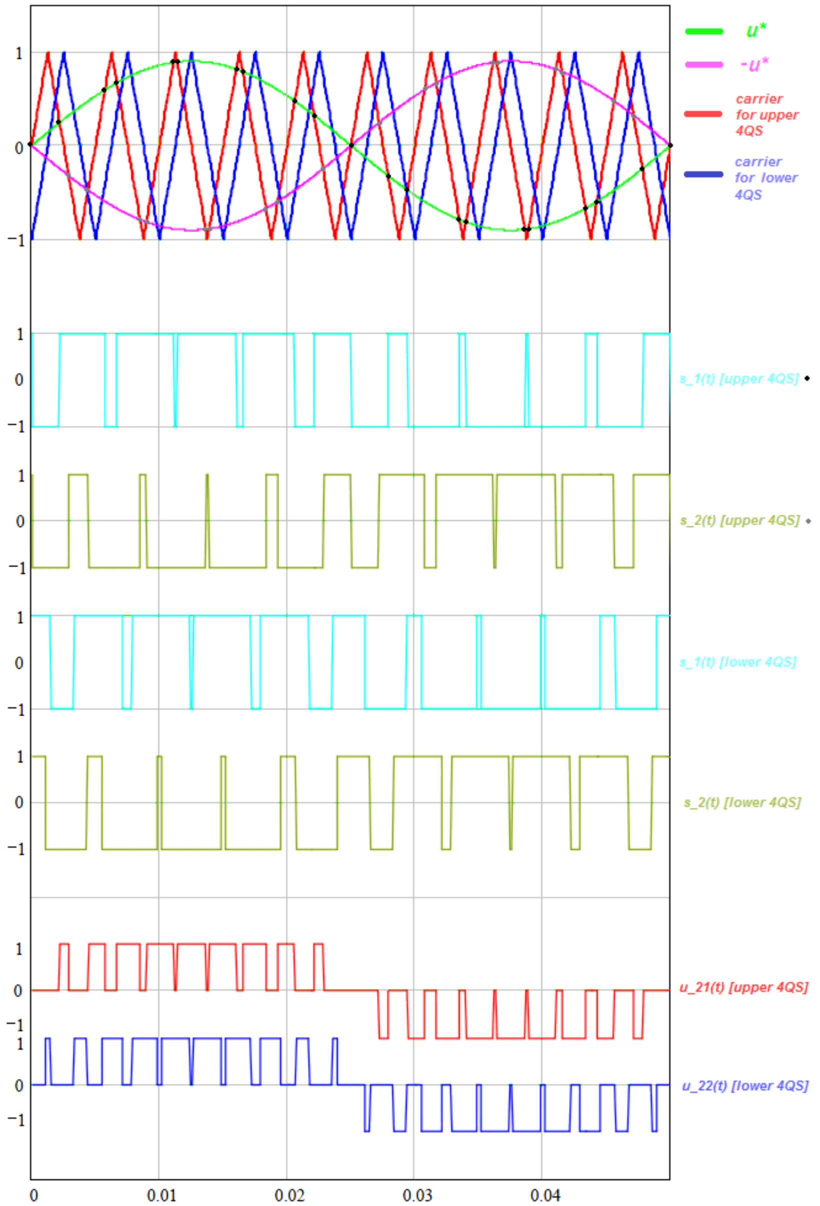
Figure 3: Power supply for an electric locomotive

Nominal output voltage	$U_{\mathrm{DC}} = 1.8\ \mathrm{kV}$	Grid voltage	$U_{\mathrm{AC}} = 25\ \mathrm{kV}$
Nominal output power	$P_{\mathrm{N}} = 2.4\ \mathrm{MW}$	Grid frequency	$f = 50\ \mathrm{Hz}$
Transformer's nominal apparent power	$S_{\mathrm{N}} = 2.6\ \mathrm{MVA}$	Switching frequency	$f_{\mathrm{S}} = 500\ \mathrm{Hz}$
Transformer's short circuit voltage (each secondary)	$u_{\mathrm{k}} = 19\%$		

3.1 Draw the circuit diagram of a four-quadrant converter with the semiconductors treated as ideal switches.



3.2 Draw qualitatively the switching functions of the 4QC-switches of the two parallel four-quadrant converters in interleaving operation. Draw the transformer secondary voltages resulting from the two four-quadrant converters for a fundamental period.



3.3 Calculate the leakage inductances of the transformer. Assume that the primary leakage inductances can be neglected.

$$u_k = \frac{U_k}{U_{AC}} 100\% \rightarrow U_K = \frac{u_k}{100\%} U_{AC}$$

$$I_{21} = \frac{S_N}{2U_{AC}}$$

$$Z_k = \frac{U_k}{I_{21}} = \frac{u_k \cdot U_{AC} \cdot 2\,U_{AC}}{100\% \cdot S_n} = \frac{19\% \cdot 2 \cdot (25\text{ kV})^2}{100\% \cdot 2.6\text{ MVA}} = 91.346\ \Omega$$

$$Z_k = \omega(\cancel{L_{\sigma p}} + L_{\sigma s}') \rightarrow L_{\sigma s}' = \frac{Z_k}{2\pi f} = \frac{91.346\ \Omega}{2\pi \cdot 50\text{ Hz}} = 0.29\text{ H} \quad (L_{\sigma p}\text{ was negeclected})$$

3.4 The two four-quadrant converters should be controlled that way that only active power is supplied from the grid (target power factor mode). Determine the phase angle of the required fundamental phasors of u_{21}, u_{22}. Consider the case of nominal load at nominal input voltage.

$$\varphi = \arctan\left(\frac{\omega L_{\sigma s}' \cdot I_{21}}{U_{AC}}\right) = \arctan\left(\frac{\omega\, L_{\sigma s}' \cdot P_n}{2\,U_{AC}^2}\right)$$

$$= \arctan\left(\frac{\pi f\, L_{\sigma s}' \cdot P_n}{U_{AC}^2}\right) = \arctan\left(\frac{50\text{ Hz } \pi\, 0.29\text{H} \cdot 2.4\text{ MW}}{(25\text{ kV})^2}\right) = \arctan\ (0.1749) = 9.922°$$

3.5 Calculate the minimum required transformer winding ratio $\alpha = \frac{N_1}{N_2}$ for the case of 3.4.

$$\alpha = \frac{U_{AC} \cdot \sqrt{2}}{\hat{U}_{S\,AC}} = \frac{\sqrt{2} \cdot 25\text{ kV}}{1.773\text{ kV}} = 19.94$$

$$\hat{U}_{S\,AC} = \hat{U}_{21}\ \cos(\varphi) = U_{DC}\ \cos(\varphi) = 1.8\text{ kV} \cdot \cos(9.922) = 1.773\text{ kV}$$

$$L_{\sigma s} = \frac{L_{\sigma s}'}{\alpha} = 0.73\text{ mH}$$

3.6 Calculate the peak voltage stress in the transistors and the diodes of the 4QC. Explain in words what has to be done to calculate the peak current stress (calculation not mandatory).

$$\hat{U}_{21} = \frac{U_{AC} \cdot \sqrt{2}}{\alpha \cdot \cos(\varphi)} = \frac{25\text{ kV} \cdot \sqrt{2}}{19.94 \cdot \cos\ (9.922)} = 1800\text{ V} = 1.8\text{ kV}$$

$$U_{DC} = 1.8\text{ kV}$$

$$I_{peak} = \underbrace{I_{12} \cdot \sqrt{2} \cdot \alpha}_{\text{Secondary side peak current}} + \underbrace{\tfrac{1}{2}\,\Delta I_2}_{\text{Current ripple}}$$

Task 4: Line-Commutated Converter (25 Points)

A line-commutated converter in M2-configuration supplies a DC-motor with a well smoothed armature current shown in figure below. The line frequency is 50 Hz.

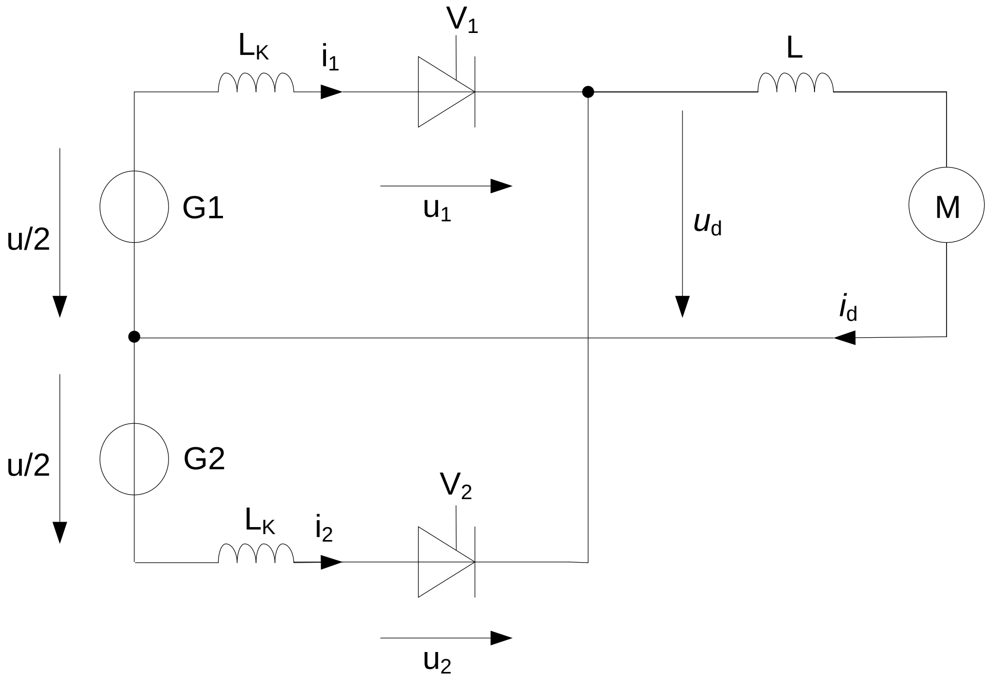
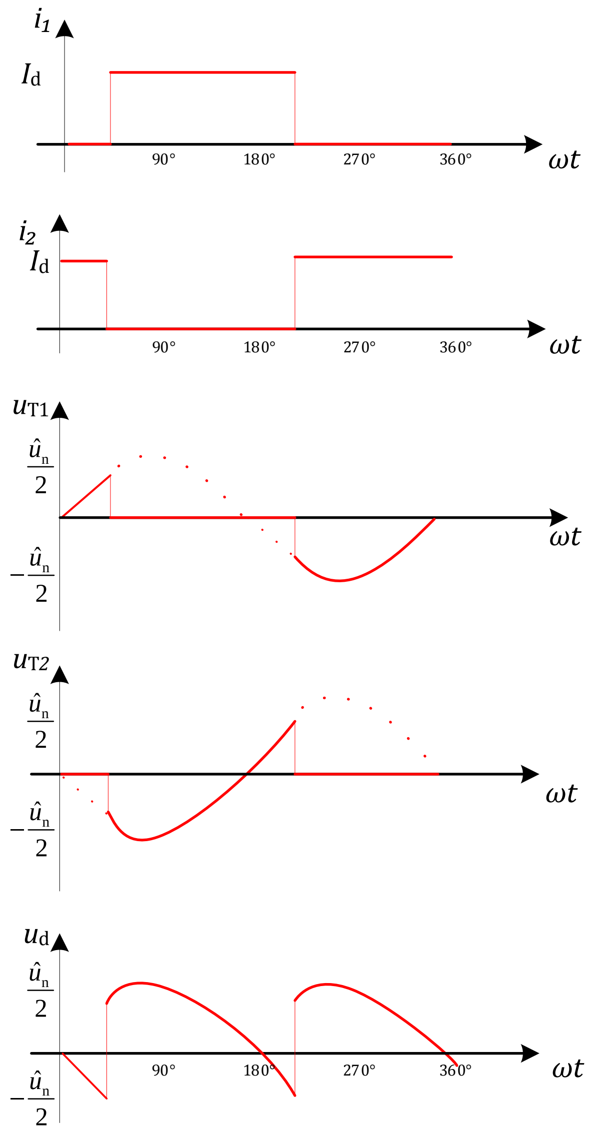


Figure 4: Line-commutated converter in M2-configuration

4.1 Sketch qualitatively the resulting waveforms for thyristor currents i_1, i_2, the thyristor voltages u_{T1}, u_{T2} and the output voltage u_d for a control angle of α = 45° assuming ideal commutations.



4.2 Derive a formula for the current i_1 during commutation for $= 0$

With commutational voltage , $u_k = U\sqrt{2}\,\sin(\omega t) = \frac{u}{2} + \frac{u}{2} = u$

Kirchhoff's current law, $i_1 + i_2 = I_d$ for the equivalent circuit

and Faraday's law, $u_k = 2\,L_K\,\frac{di_K}{dt}$

we obtain $i_k = \frac{1}{2\,L_K}\int u_K\,dt = -\frac{U}{\sqrt{2}\,\omega L_K}\cos(\omega t) + C,$

with initial conditions $i_{K_{\omega t=\alpha}} = 0$

follows $C = \frac{u}{\sqrt{2}\,\omega L_K}\cos\alpha$

Hence, we obtain $i_k\ (t) = \frac{U}{\sqrt{2}\,\omega L_K}\ (\cos(\alpha) - \cos(\omega t)) = i_1(t),\ if\ \omega t = \ \alpha$ (firing angle of T_1)

and its max. value $\hat{\imath}_k\ = \frac{\hat{U}}{2\ \omega L_k}$

The initial overlapping angle κ_θ (often denoted u_θ in literature) results from the relation $i_{k_{\omega t = \alpha + U}} = I_d$ wherefrom $i_{1_{\omega t = \alpha + \kappa}} = I_d$ and $i_{2_{\omega t = \alpha + \kappa}} = 0$

$$\kappa = u_\theta = cos^{-1}\left(\cos(\alpha) - \frac{I_d\sqrt{2}\,\omega L_K}{U}\right) - \alpha$$

$$= cos^{-1}\left(1 - \frac{I_d}{\hat{\imath}_k}\right) - \alpha = 26°$$

4.3 Calculate the inductive voltage drop due to the commutation.

During the current commutation interval K both thyristors conduct simultaneously and the phase voltages are shorted by L_k in each phase. The current builds up in the thyristors switching on from $0\ to\ I_d$, whereas the current being switched-off during overlapping time decays from I_d to 0. The reduction in volt-radians area during one commutation is

$$A_c = \int_\alpha^{\alpha+K} u_{LK}\ d(\omega t) = \omega L_K\ \int_0^{I_d} di = \omega L_K I_d$$

The average DC-voltage loss is then $\frac{2A_c}{\pi}$ or in general

$$D_x = pf L_k I_d = 100\ \frac{1}{\mathrm{s}}\cdot 1\cdot 10^{-3}\ \frac{\mathrm{Vs}}{\mathrm{A}}\cdot 50\ \mathrm{A} = 5\ \mathrm{V}$$

4.4 Calculate the maximum control angle α_{max}, if the commutating inductors are $L_K\ = 1\ \mathrm{mH}$, the output current is $I_d\ =\ 50\ \mathrm{A}$, the recovery time of the thyristors to block the recurring positive voltage is $t_c\ =\ 300\mu\mathrm{s}$ and the line-to-line voltage amplitude is 400 V.

Using the relation derived for $i_k\ (t)$ in task 4.2:

$i_k = \frac{U}{\sqrt{2}\,\omega L_K}\ (\cos\alpha - \cos(\omega t)$ and the definition of the extinction angle $\gamma = \pi - (d + \kappa)$ and knowing that $\gamma \overset{!}{<} \omega t_c$ to ensure a proper commutation at maximum operation in the inverter mode of operation we are able to formulate:

$\alpha + \kappa + \gamma = \pi.$ Substituting this into $i_k(t)$ we obtain:

$$I_d = \frac{U}{\sqrt{2}\,\omega L_K}(\cos(\alpha) - \cos(\alpha + \kappa))$$
$$= \frac{U}{\sqrt{2}\,\omega L_K}(\cos(\alpha_{max}) - \cos(\pi - \gamma))$$

Hence α_{max}results as: $\alpha_{max} \leq cos^{-1}\left[\frac{I_d\sqrt{2}\,\omega L_K}{U} + \cos\ (\pi - \omega t_c)\right] = 153.5\ °$