

Musterlösung - Bonustest 1 - 2008

Aufgabe 1: Mittelwert und Effektivwert

1.1)

$$i_R(\omega t) = \begin{cases} \frac{u(t)}{R} & 0 \leq \omega t < \pi \\ 0 & \pi \leq \omega t < 2\pi \end{cases} = \begin{cases} 4A \cdot \sin(\omega t) & 0 \leq \omega t < \pi \\ 0 & \pi \leq \omega t < 2\pi \end{cases}$$

1.2)

$$\begin{aligned} \overline{i_R} &= \frac{1}{2\pi} \int_0^{2\pi} i_R(\omega t) d\omega t = \frac{1}{2\pi} \int_0^{\pi} 4A \cdot \sin(\omega t) d\omega t \\ &= -\frac{4A}{2\pi} \cdot \cos \omega t \Big|_0^{\pi} = \frac{2A}{\pi} \cdot 2 = \frac{4}{\pi} A \approx 1,273A \end{aligned}$$

1.3)

$$\begin{aligned} I_R &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_R^2(\omega t) d\omega t} \\ I_R^2 &= \frac{1}{2\pi} \int_0^{\pi} 16A^2 \cdot \sin^2(\omega t) d\omega t \\ &= \frac{8A^2}{\pi} \int_0^{\pi} \sin^2 \omega t d\omega t = \frac{8A^2}{\pi} \left[\frac{\omega t}{2} - \frac{1}{4} \sin 2\omega t \right]_0^{\pi} = 4A^2 \\ I_R &= 2A \end{aligned}$$

1.4)

$$\begin{aligned} k_s &= \frac{\hat{i}_R}{I_R} = \frac{4A}{2A} = 2 \\ k_f &= \frac{I_R}{|\overline{i_R}|} = \frac{I_R}{\overline{i_R}} = \frac{\pi}{2} \approx 1,57 \end{aligned}$$

1.5)

$$P = \overline{p} = \frac{I_R^2}{R} = \frac{(2A)^2}{1,5\Omega} \approx 2,666W$$

Aufgabe 2: Kapazitäten

2.1)

$$C_g = C_1 + C_2 = 20\text{mF}$$
$$u(t) = \frac{1}{C} \int_0^t i(\tilde{t}) d\tilde{t} + u(0) = \begin{cases} \frac{3\text{A}}{20\text{ms}} \cdot t & 0 \leq \omega t < 1\text{ms} \\ -\frac{1,5\text{A}}{20\text{ms}} \cdot t + \frac{3\text{A} \cdot 1\text{ms}}{20\text{ms}} & 1\text{ms} \leq \omega t < 2\text{ms} \\ \frac{1,5\text{A}}{20\text{ms}} \cdot t + \frac{1,5\text{A} \cdot 1\text{ms}}{20\text{ms}} & 2\text{ms} \leq \omega t < 3\text{ms} \\ -\frac{3\text{A}}{20\text{ms}} \cdot t + \frac{3\text{A} \cdot 1\text{ms}}{20\text{ms}} & 3\text{ms} \leq \omega t < 4\text{ms} \end{cases}$$
$$= \begin{cases} 0,15 \frac{\text{V}}{\text{ms}} \cdot t & 0 \leq \omega t < 1\text{ms} \\ -0,075 \frac{\text{V}}{\text{ms}} \cdot t + 0,15\text{V} & 1\text{ms} \leq \omega t < 2\text{ms} \\ 0,075 \frac{\text{V}}{\text{ms}} \cdot t + 0,075\text{V} & 2\text{ms} \leq \omega t < 3\text{ms} \\ -0,15 \frac{\text{V}}{\text{ms}} \cdot t + 0,15\text{V} & 3\text{ms} \leq \omega t < 4\text{ms} \end{cases}$$

2.2)

$$w(t) = \frac{1}{2} C_g u^2(t)$$
$$w(t_1) = \frac{1}{2} \cdot 20\text{mF} \cdot (0,15\text{V})^2 = 0,225\text{mJ}$$
$$w(t_2) = \frac{1}{2} \cdot 20\text{mF} \cdot (0,075\text{V})^2 = 0,05625\text{mJ}$$

2.3)

$$i_{C_2} = C_2 \cdot \dot{u}(t) = C_2 \cdot \frac{d}{dt} \left(\frac{1}{C_g} \cdot \int i(t) dt + u(0) \right) = \frac{C_2}{C_g} \cdot i(t) = \frac{C_2}{C_1 + C_2} \cdot i(t)$$

2.4)

$$p(t) = u(t) \cdot i(t)$$

