Design of Drives with Optimum Load Acceleration

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Abstract—In this paper the product of torque and acceleration ($M\alpha$-product) is used for design of drives at which load acceleration has to be optimized. At optimization of such drives this product plays the same role that mechanical power (product of torque and speed) plays at the design of constant-speed drives because both products are transmitted unchanged by an ideal gear. By use of $M\alpha$-product design procedure becomes very clear and consideration of steady-state load torque is very simple. Last not least, power rate, a motor constant characterizing its capability to accelerate inertial loads, proves to be the value of $M\alpha$-product which the motor generates during no-load acceleration.

I. INTRODUCTION

Design of drives for constant speed applications at which torque required for acceleration is negligible is simple because mechanical power is transmitted unchanged by an ideal gear. Consequently each motor is suitable for an application if the rated power of the motor is more than the product of torque and speed required by the load. Normally torque and speed of the motor will not match the requirement of the load but this problem can be solved by use of a gear.

This design procedure can also be applicable if the percentage of torque required for acceleration of load inertia is low. But with increasing acceleration torque this method becomes ineffective because it does not consider the portion of torque which is required to accelerate the inertia of the motor and is not available for the load. When looking for motor data which depend on the motor inertia the no-load acceleration of the motor can be found in data sheets of many servo motors. But this motor constant decreases with increasing torque and size of motors so it is obviously useless for selection of motors.

The motor constant characterizing a motor's capability to accelerate inertial loads is the power rate. This quantity is known since long [1, 2] but not well-known. In the following sections this quantity will be recalled. In addition the use of power rate is extended to drive applications at which the load torque required at constant speed cannot be neglected.

The suggested design method is based on a close analogy existing between the design of constant-speed drives and accelerating drives at which the product of torque and acceleration ($M\alpha$-product) takes the place of mechanical power ($M\omega$-product). Therefore the design of constant-speed drives will be described shortly in Section II before the design of acceleration drives and the meaning and importance of the $M\alpha$-product is discussed in Section III.

II. DESIGN OF CONTINUOUS SPEED DRIVES

Design of continuous-speed drives starts from the nominal values of (steady-state) load torque M_{Ln} and speed ω_{Ln} from which power requirement of the load P_{Lerf} is calculated. For a given application all motors are suitable the nominal power P_{Mn} of which is at least as much as this value,

$$P_{Mn} \geq P_{Lreq} = M_{Ln} \cdot \omega_{Ln} \,. \tag{1}$$

If torque and speed of the motor do not match the related requirements of the load they can be adapted by use of a gear. If the gear is assumed to have no losses it is characterized only by its gear ratio i_G and torques and speeds of motor and load are related by

$$M_{GL} = M_{MG} \cdot i_G \,, \qquad \omega_L = \frac{\omega_M}{i_G} \,, \tag{2}$$

where M_{MG} und M_{GL} are the shaft torques applied by the motor to the gear and by the gear to the load, respectively. ω_M and ω_L are the motor-side and load-side speed, see Fig. 1.

Normally the nominal power of suitable motors will not match the required power exactly but will be greater. In this case calculation of gear ratio either from torques or from speeds delivers different results,

$$i_{G1} = \frac{M_{Ln}}{M_{Mn}} \,, \qquad i_{G2} = \frac{\omega_{Mn}}{\omega_{Ln}} \,. \tag{3}$$

Gear ratio can now be chosen from inside the range

$$i_{G1} \leq i_G \leq i_{G2} \tag{4}$$

where the limits $i_{G1} \leq i_{G2}$ result from $M_{Mn}\,\omega_{Mn} > M_{Ln}\,\omega_{Ln}$.

The transformation of the motor's torque and speed by the gear and the effect of different gear ratios are visualized by $M\omega$-diagrams at Fig. 2 and Fig. 3.

The working basis is presented at Fig. 2. Load characteristic l shows the dependence of load-side speed ω_L and shaft torque M_{GL} required by the load. Since speed is considered constant shaft torque matches the resistive load

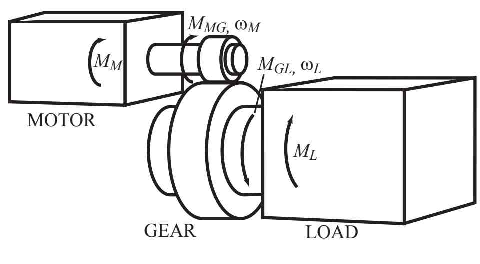

Fig. 1: Structure of drive consisting of motor, gear and load

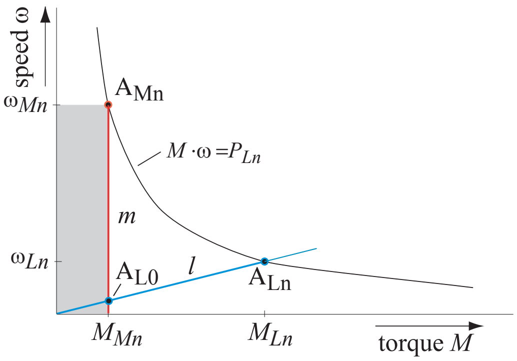

Fig 2: Characteristics *l* of load and *m* of motor

torque M_L which can be braking (M_L >0) or driving (M_L <0). The thick part of the characteristic on which the load must be operated is limited by its nominal operating point A_{Ln} at which maximum power P_{Lreq} is required.

Furthermore a motor characteristic *m* can be seen showing the relation between the motor's shaft torque M_{MG} and speed. Since the torque M_M generated by the motor is assumed to match its nominal value M_{Mn} characteristic *m* limits the shaded operation range of the motor. As can be seen the operation range does not contain but a fraction of the load characteristic; the motor cannot drive the load beyond operating point A_{L0} because of lacking torque. On the other hand the motor has a great reserve of speed and power. It is able to deliver the power required by the load because its nominal operating point A_{Mn} and the critical operating point of the load A_{Ln} are situated exactly on the same hyperbola $Mw = P_{Ln}$. Consequently A_{Mn} can be shifted to A_{Ln} by applying a gear having the gear ratio $i_G = i_{G1} = i_{G2}$.

At Fig. 3 torque characteristic *g* of the geared motor is shown which is obtained when the motor is generating its nominal torque. The related operation region of the drive is shaded at the figure. The latter includes load characteristic *l* totally and operating point A_{Gn}, at which the geared motor delivers its maximum power P_{Ln}, is situated exactly on the nominal operating point A_{Ln} of the load. From the figure can be seen, too, that the characteristics *l*, *m* and *g* must not necessarily be straight lines, but they can have any shape as far as the load characteristic does not leave the operating region of the geared motor.

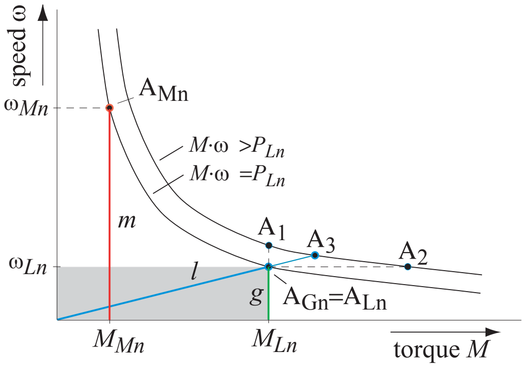

Fig 3: Speed-torque characteristics *l* of load, *m* of motor and *g* of geared motor

Fig. 3 is also used to visualize the possibilities for the choice of gear ratio if the nominal power of the motor is greater than the power required by the load. In this case A_{Mn} is situated somewhere on a hyperbola $Mw > P_{Ln}$ and three possibilities for the choice of gear ratio are of special interest:

- If the gear ratio is chosen according to torque ratio i_{G1} the nominal operating point of the motor is shifted to A_1. Hence the torque requirement of the load is fulfilled exactly and, considering the requirements of the load at A_{Ln}, a reserve of speed exists.
- If the torque gear ratio is chosen according to speed ratio i_{G2} the nominal operating point of the motor is shifted to A_2. Hence the speed requirement of the load is fulfilled exactly and a reserve of torque has been achieved.
- In case the load characteristic is exactly linear as assumed at the figure a reserve of torque and speed is required if the operation region of the load shall be extended. It is easy to prove that the load characteristic can be utilized up to point A_3 if the gear ratio is chosen according to

$$i_{G3} = \sqrt{\frac{M_{Ln}}{M_{Mn}} \cdot \frac{\omega_{Mn}}{\omega_{Ln}}} = \sqrt{i_{G1} \cdot i_{G2}} \ . \tag{5}$$

III. DESIGN OF DRIVES FOR HIGH ACCELERATION

Analysis of acceleration drives, from which design equations will be derived, starts from equations showing how the acceleration α_M of the motor-side inertia J_M and the acceleration α_L of the load-side inertia J_L depend on the respective internal torques (M_M, M_L) and shaft torques (M_{MG}, M_{GL}),

$$J_M \cdot \alpha_M = M_M - M_{MG} \tag{6}$$

$$J_L \cdot \alpha_L = M_{GL} - M_L \ . \tag{7}$$

In contrast with the relations existing with continuous-speed drives shaft torques of motor and load do not agree with the internal torques because torque required for acceleration of inertias cannot be neglected.

If the inertia of the gear is neglected in addition to losses (or if it is included in J_M and J_L) the motor-side and load-side shaft-torques and speeds are related by

$$M_{GL} = M_{MG} \cdot i_G \ , \qquad \alpha_L = \frac{\alpha_M}{i_G} \ . \tag{8}$$

Note that the relation for acceleration agrees closely with relation for speeds because accelerations are derived from speeds by differentiation.

From (8) an important result can be achieved

$$\alpha_L M_{GL} = \alpha_M M_{MG} \ . \tag{9}$$

This means that the product $Q=M\alpha$ is transferred by an ideal gear just like the product $P=M\omega$. This finding will be the base for the suggested design procedure.

When the shaft torques are eliminated from (6) and (7) by use of (8) load acceleration results as

$$\alpha_L = \frac{i_G \cdot M_M - M_L}{J_L + i_G^2 \cdot J_M} \ . \tag{10}$$

In this equation the nominator of the right-hand side represents the difference of external torques affecting the system while the denominator consists of the load-side and motor-side inertia. All quantities are referred to the load side.

In the following sections calculation of gear ratio will deliver square roots which can be positive or negative. Considering that a negative gear ration does not mean but an opposite direction of the load-side motion, only positive values of gear ratio will be given as well as positive values of load acceleration which result from these gear ratios.

A. *Design of drives with pure inertial load*

The problem to be investigated becomes simple when load torque M_L is neglected. For this case Fig. 4 shows how load acceleration depends on gear ratio. Calculation of extreme values delivers (e.g. see [3]) that optimum load acceleration

$$\alpha_{Lopt} = \frac{M_M}{2 \cdot \sqrt{J_L \cdot J_M}} \tag{11}$$

is achieved by realization of the optimum gear ratio

$$i_{Gopt} = \sqrt{\frac{J_L}{J_M}} \,. \tag{12}$$

Inserting (12) into (10) proves that in this case inertias of motor and load are equal when referred to the same side of the gear, $J_L = i_{Gopt}^2 \cdot J_M$. Furthermore, from (11) can be seen that, as far as the motor is concerned, the achievable load acceleration is determined by

$$Q_{M\,\max} = \frac{M_{M\,\max}^2}{J_M} \,. \tag{13}$$

This fact is known since long and in English literature motor constant Q_{Mmax} is called power rate [1]. Normally its value is based on the maximum torque M_{Mmax} which is permitted for a specified limited time which acceleration must not exceed.

By use of power rate optimum load acceleration can be given as a function of only one motor constant

$$\alpha_{Lopt} = \frac{1}{2} \cdot \sqrt{\frac{Q_{M\,\max}}{J_L}} \,. \tag{14}$$

For drive design (14) can be arranged for calculation of the power rate which the motor must have for being suitable to achieve the load acceleration demanded by the application:

$$Q_{M\,\max} \geq Q_{Mreq} = 4 \cdot J_L \cdot \alpha_{Lreq}^2 \,. \tag{15}$$

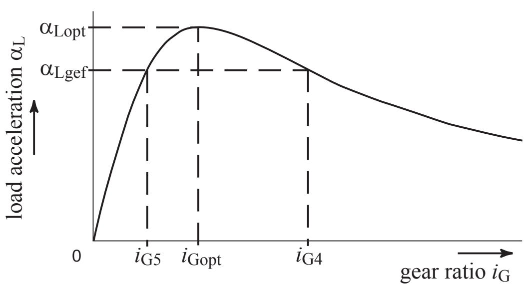

Fig. 4: Load acceleration achieved at constant motor torque depending

As it was the case with continuous speed drives the capability of suitable motors will be greater than required. Then again gear ratio can be chosen from within a limited range $i_{G5} \leq i_G \leq i_{G4}$, see fig. 4. The limits result from

$$\alpha_L / \alpha_{Lopt} = \frac{2 \cdot i_G / i_{Gopt}}{1 + (i_G / i_{Gopt})^2} \,, \tag{16}$$

which is achieved by inserting (11) and (12) into (10). If α_L is replaced α_{Lreq} by we get from (16)

$$i_{G4,5} = i_{Gopt} \cdot (\alpha_{Lopt} / \alpha_{Lreq} \pm \sqrt{(\alpha_{Lopt} / \alpha_{Lreq})^2 - 1}) \tag{17}$$

or

$$i_{G4,5} = i_{Gopt} \cdot (\sqrt{Q_{M\,\max} / Q_{Mreq}} \pm \sqrt{Q_{M\,\max} / Q_{Mreq} - 1}) \,. \tag{18}$$

Now drive optimization shall be visualized by diagrams as already done for continuous speed drives. But speed w will now be replaced by acceleration a.

At Fig. 5 torque-acceleration characteristics *l* of load and *m* of speed can be seen. Characteristics being in accordance with (6) and (7) are principally linear; their slopes are determined by the corresponding inertias.

- Load characteristic *l* shows the dependence of load acceleration a_L on the load's shaft torque M_{GL} and intersects the torque axis at the value of load torque M_L (here $M_L = 0$). The range in which the load must be operated is marked by point A_{Lreq} at which maximum acceleration is required.
- Motor characteristic *m* shows acceleration a_M depending on the loading shaft torque M_{MG} in case maximum torque M_{Mmax} is generated by the motor. Torque axis is intersected at exactly this value while acceleration axis is intersected at the related no-load acceleration of the motor. Characteristic *m* limits the shaded region in which the motor can be operated without exceeding the maximum torque.

The acceleration a_0 achievable without a gear (special case $i_G = 1$) can be read from the intersection A_0 of both characteristics; obviously it is much smaller than required. Nevertheless the motor is suitable for this application as can

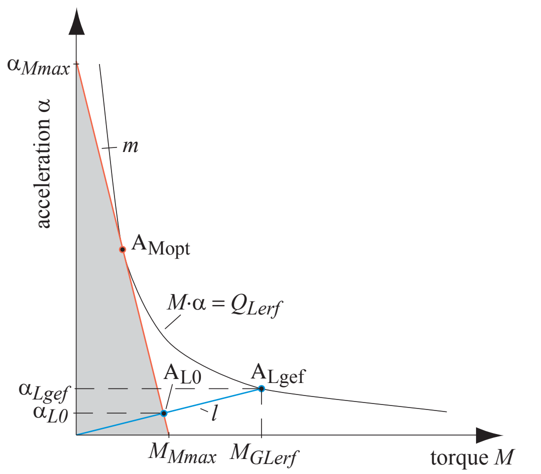

Fig. 5: Acceleration-torque characteristics *l* of load and *m* of motor

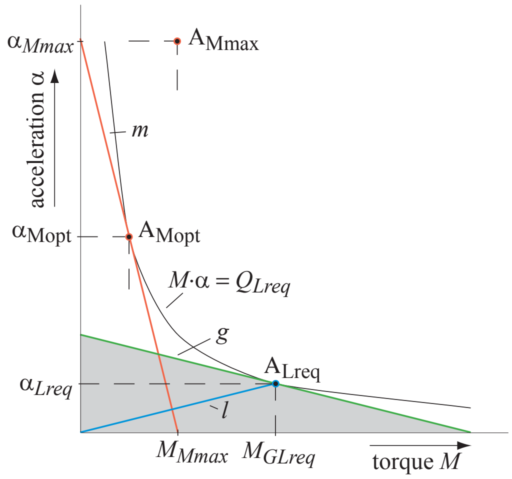

Fig. 6: Acceleration-torque characteristics *l* of load, *m* of motor and *g* of geared motor

be proved by consideration of the product $Q=M\alpha$ which is transmitted unchanged by an ideal gear, see (8). And in fact, there is one point A_{Mopt} on the motor's characteristic *m* at which the $M\alpha$-product delivered by the motor (Q_{Mopt} = $M_{MGopt}\alpha_{Mopt}$) is as much as $M\alpha$-product required by the load ($Q_{Lreq}= M_{Lreq}\ \alpha_{Lreq}$). Hence the motor can apply the demanded load acceleration when geared with the optimal gear ratio i_{Gopt} by which the motor's optimal operating point A_{Mopt} is shifted to the critical operating point A_{Lreq} of the load.

The result of the optimization is shown at Fig. 6. Characteristic *g* of the geared motor meets operating point A_{Lreq} of the load which now can be operated as required.

B. Power rate and $M\alpha$-product

From another notation of power rate,

$$Q_{M\max} = \frac{M^2_{M\max}}{J_M} = M_{M\max} \cdot \alpha_{M\max}\,, \tag{19}$$

can be seen that power rate, being the product of maximum motor torque and related no-load acceleration α_{Mmax}, is a special value of the $M\alpha$-product appearing at no-load operation.

Equation (19) can also be transformed to

$$Q_{M\max} = M_{M\max}\omega_{Mn} \cdot \frac{\alpha_{M\max}}{\omega_{Mn}} = \frac{P_{M\max}}{T_{Mh}}\,. \tag{20}$$

In this expression P_{Mmax} and T_{Mh} represent the motor's maximum power and minimum acceleration time generated at maximum torque. Obviously power rate is the gradient of generated power during acceleration of the no-loaded motor which explains the name "power rate" as well as its unit W/s (usually kW/s).

Diagrams given above have already made plain that the $M\alpha$-product has the same importance and a similar physical meaning for acceleration drives as the $M\omega$ -product has for continuous-speed drives. And in fact the following investigation of $M\alpha$-product $Q = M$a will make possible to consider load torque without further calculation of extreme values:

Design rule (15) can be obtained from the following steps:

- When unloaded motor is accelerated by its maximum torque it generates the maximum $M\alpha$-product $Q=Q_{Mmax}$ which equates the motor's power rate. The related point in the $M\alpha$-diagram is determined by M_{Mmax} and α_{Mmax}, see A_{Mmax}, see Fig. 6.
- When the motor is coupled to the load with the optimal gear ratio the total inertia to be accelerated by the motor is doubled. Consequently the generated $M\alpha$-product is reduced to one half of the genuine value, $Q_M = M_{Mmax}\ \alpha_{Mopt} = Q_{Mmax}/2$.
- Since one half of the generated torque is used to accelerate the motor's inertia, only one half is applied to the shaft. Hence the $M\alpha$-product applied to the gear becomes $Q_{MG} = M_{MG}\ \alpha_{Mopt}$ $=Q_{Mmax}/4$. This result is confirmed by Fig. 6 where the ratio of the areas spanned by operating points A_{Mopt} and A_{Mmax} is 1:4.
- By an ideal gear $M\alpha$-product Q_{MG} is transmitted without losses. Consequently the $M\alpha$-product applied to the load is

$$Q_{GL} = M_{GL}\alpha_L = \frac{Q_{M\max}}{4}\,. \tag{21}$$

$M\alpha$-product applied to the load must match $M\alpha$-product consumed by the load. For an inertial load the consumed $M\alpha$-product is calculated considering $M_{GL} = J_L\ \alpha_{Lreq}$

$$M_{GL}\alpha_L = J_L\alpha_L^2 = \frac{Q_{M\max}}{4}\,. \tag{22}$$

Note that this equation is equivalent to design rule (15) if the required acceleration α_{Lreq} is inserted.

C. Consideration of load torque M_L

Optimization of load acceleration under considerations of load torque can be performed, of course, in the conventional way i.e. by calculating extreme values from (10). But without computer aid calculation is troublesome and, whether computers are used or not, results are badly arranged and difficult to interpret. A new finding will be presented now by which optimal gear ratio and load acceleration are determined only by use of already achieved knowledge and without calculation of extreme value.

$M\alpha$-product applied to the load, see (21), was derived without introducing any assumption concerning the load. Consequently (21) must also hold when load torque M_L is considered. In this case required $M\alpha$-product becomes

$$Q_{GL} = M_{GL}\alpha_L = (J_L \cdot \alpha_L + M_L) \cdot \alpha_L\,. \tag{23}$$

When the $M\alpha$-products applied to and consumed by the load are equated a relation is achieved

$$J_L \cdot \tilde{\alpha}^2_{Lopt} + M_L \cdot \tilde{\alpha}_{Lopt} = \frac{Q_{M\max}}{4} \tag{24}$$

from which optimal acceleration $\tilde{\alpha}_{Lopt}$, achievable under consideration of load torque M_L, can be calculated as

$$\tilde{\alpha}_{Lopt} = \alpha_{Lopt} \cdot \left(\sqrt{1 + \left(\frac{M_L}{2J_L\alpha_{Lopt}} \right)^2} - \frac{M_L}{2J_L\alpha_{Lopt}} \right). \tag{25}$$

At drive design the required power rate is of interest for selection of a suitable motor. It results from rearranging (24),

$$Q_{M\max} \geq Q_{Lreq} = 4 \cdot J_L \tilde{\alpha}_{Lreq}^2 \cdot (1 + \frac{M_L}{J_L \tilde{\alpha}_{Lreq}}) . \qquad (26)$$

In accordance with this equation an equivalent load inertia

$$\tilde{J}_L = J_L \cdot (1 + \frac{M_L}{J_L \tilde{\alpha}_{Lreq}}) \qquad (27)$$

can be defined. This quantity can be used to apply design equations (11), (12) and (15), derived for pure inertial loads, to systems at which load torque must be considered. But expression achieved from (12) for $\tilde{i}_{Gopt}$ is complicate.

Therefore another method for determination of optimal gear ratio is presented now which is also based on the above consideration of $M\alpha$-product.

It has already been established that, with optimal gear ratio, the motor-side acceleration is half the no-load acceleration. Consequently the optimal gear must transform the load acceleration exactly to this acceleration whether a load torque is considered or not. Therefore

$$\tilde{i}_{Gopt} \cdot \tilde{\alpha}_{Gopt} = i_{Gopt} \cdot \alpha_{Gopt} = \alpha_{M\max}/2 . \qquad (28)$$

can be used to determine $\tilde{i}_{Gopt}$ from $\tilde{\alpha}_{Gopt}$ and $\alpha_{M\max}$. The result

$$\tilde{i}_{Gopt} = i_{Gopt} \cdot \left(\sqrt{1 + \left(\frac{M_L}{2 J_L \alpha_{Lopt}} \right)^2} + \frac{M_L}{2 J_L \alpha_{Lopt}} \right) \qquad (29)$$

obtained is closely similar to (25) for $\tilde{\alpha}_{Gopt}$.

At Fig. 7 $\tilde{\alpha}_{Gopt}$ and $\tilde{i}_{Gopt}$ are plotted as functions of the load torque which can be braking (M_L >0) or driving (M_L <0). It is worth mentioning that even very high braking torques will not cause negative acceleration because the motor's torque is always sufficiently amplified by a great gear ratio due to the optimization of load acceleration.

Concluding this section it should be mentioned that different gear ratios result from (29) for braking torque (M_L >0) or driving torque (M_L <0). Furthermore load torque can vary depending on speed as assumed at Fig. 2. Therefore a decision has to be made under what condition optimal acceleration shall be obtained (if the gear ratio cannot be varied during operation).

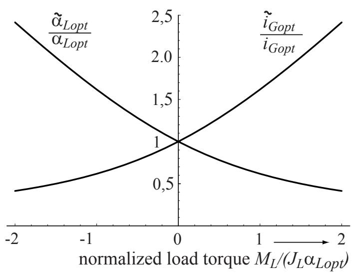

Fig. 7: Optimal values of normalized load acceleration and gear ratio depending on normalized torque

IV. Design considering Steady-state Operation and Acceleration Processes

When designing a drive it is important to ensure that the requirements of steady-state operation and acceleration processes are fulfilled by motor and gear simultaneously. This means that on the one hand nominal power and power rate of the motor must be suffiently high. On the other hand the gear ratio, which depends on data of motor and load, must agree with the demands of steady-state operation $i_{G1} \leq i_{Gopt} \leq i_{G2}$ and acceleration processes $i_{G5} \leq i_{Gopt} \leq i_{G4}$. Consequently only those motors are suitable for which these ranges have an overlap or, with other words, which have a reserve of power and/or power rate. The best suitability for a given load, of course, will have a motor for which the ranges and the related reserves of power and power rate are smallest.

But reserves of power and power rate are also required due to other reasons. Design of drives has been discussed based on an ideal gear. Losses caused by friction and slip have been neglected and the contribution to the motor-side and load-side inertia cannot be known in advance because they depend on the size of the gear which is determined by its transmission ratio. This is why the design should be checked finally under consideration of the neglected effects.

Considerations and results achieved in this paper have a wide range of application: they do not depend on the motor technology (electric, hydraulic or pneumatic) and they can be easily applied to linear loads and/or motors by simply replacing torques by forces, angular speeds by linear speeds and inertia by masses.

V. Conclusion

After acceleration of pure inertial loads had been investigated in a conventional way the optimization was visualized in $M\alpha$-diagrams. $M\alpha$-product, which is transmitted unchanged by an ideal gear like the mechanical power, proved to be an important quantity which for acceleration processes is as important as power ($M\omega$-product) for continuous-speed operation. By use of $M\alpha$-product load acceleration was optimized under consideration of steady-state load torque in a very simple way and without calculation of extreme values.

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