The Design of Long-Stator Linear Motor Drives for RailCab Test Track

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ABSTRACT

The basic equations of a doubly-fed long-stator linear motor for a shuttle-based railway system are established. They show which degrees of freedom exist for controlling the motor. The ratio of stator and rotor current proves to be an important parameter in determining the design of motors, converters and mechanics.

Keywords: doubly-fed linear motor, energy transfer, motor design, normal force

1. Introduction

At the University of Paderborn, a new railway system called RailCab is under investigation for modernization. The following features characterize the system:\[1\]:

- Instead of trains traveling in accordance with a fixed schedule autonomous shuttles traveling on request are used for transportation of passengers and cargo. Since traveling can be performed without intermediate stops shorter times for transportation can be achieved without high-speed operation.
- On main routes, shuttles form convoys and travel at a consistent speed of about 160 km/h. When arriving at its destination a shuttle has to leave the main track for disembarking people or unloading cargo.
- While the vehicle is driven, linear motor technology is used which allows great force to be generated and steep slopes to be climbed with ease. By use of linear motors moving parts are avoided and wear of wheels is reduced remarkably.

- Undercarriages are equipped with active steering of the vehicles. This feature is required because conventional switches cannot react as quickly as necessary when a shuttle is forced off of a convoy.

In the framework of NBP (Neue Bahntechnik Paderborn) project extensive preliminary investigations have been performed and in 2003 a test track of 530 m length (scale 1:2.5, maximum slope 5.3 %) was put in operation, see Fig. 1.

![Fig. 1 Test track of NBP project](image)

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Shuttles driving on the track, see Fig. 2, with maximum speed of 10 m/s are controlled by radio frequency.

Fig. 2 Shuttle of NBP test track

For the formation of convoys, the shuttles must be operated at different speeds. Therefore asynchronous motor technology is used at the test track at which long-stator sections \(^2\) as well as short-stator \(^4\) sections are realized, see Fig. 3.

Fig. 3 Change from long-stator(left) to short-stator motor (reaction plate at right)

When driving on long-stator sections, energy can be transferred via the linear motor to the on-board power supply of the shuttle where it can be stored in batteries, visible at Fig. 2. Due to this feature neither current rails nor overhead wires are used at the test track. During operation on short-stator sections, where a reaction plate is mounted on the tracks, energy for driving must be delivered by the batteries.

This paper is devoted only to the long-stator motor. The most important aspects are discussed which determine the design of the linear drive and the strategy of feeding the motor. (For information on control and communication refer to \(^5\))

2. Assumptions

In order to simplify the investigations the following assumptions are introduced:

- The linear motor is of the doubly-fed asynchronous type at which the secondary (mounted on the vehicle) as well as the primary (stator, mounted on the track) is equipped with three-phase windings, see Fig. 4.

Fig. 4 Two shuttles with doubly-fed linear motor on long-stator motor

- The primary and secondary are fed by current-controlled inverters allowing to set amplitude, frequency and phase of the currents arbitrarily.
- Currents and voltages are assumed to be symmetrical and sinusoidal (steady-state operation) allowing to use complex phasors. Magnitudes of phasors are rms-values.
- Voltages and currents cannot be sinusoidal without the following assumptions:
  - The contributions of primary and secondary to the traveling field move synchronously i.e. with exactly the same speed. This means
    \[
    v_M + v_2 = v_1 \quad \text{or} \quad \omega_M + \omega_2 = \omega_1 \tag{1}
    \]
  - Magnetic saturation, iron losses and field harmonics are neglected.
- All kinds of boundary effects of the motors are neglected.
- All phasors are in d,q-coordinates which are oriented to the primary current. Consequently the following equations hold by definition:

\[
I_1 = I_{1d} e^{j0} \quad \text{or} \quad I_1 = I_1 + j0
\]
\[
I_2 = I_{2q} e^{j\alpha} \quad \text{or} \quad I_2 = I_{2d} + jI_{2q} \tag{2}
\]
v_M: mechanical speed of shuttle
v_{1,2}: speed of traveling wave of primary/secondary
ω_{0h}: angular frequency representing mechanical speed
ω_{1,2}: angular frequency of primary/secondary
I_{1,2}: phasor of primary/secondary current
α: phase shift between currents and magnetic fields of primary and secondary windings.

3. Basic equations

3.1 Voltage and power equations
As with rotating asynchronous machines the terminal voltages depend on the currents as follows:

\[
\begin{align*}
U_1 &= R_1 I_1 + jω_1 L_{1\sigma} I_1 + jω_1 L_{1m} I_1 + jω_1 L_{12} I_2 \\
U_2 &= R_2 I_2 + jω_2 L_{2\sigma} I_2 + jω_2 L_{2m} I_2 + jω_2 L_{12} I_1
\end{align*}
\]

(3)

\(U_{1,2} \): terminal voltage of primary/secondary
\(I_{1,2} \): current of primary/secondary
\(R_{1,2} \): resistance of primary/secondary
\(L_{1,2\sigma} \): leakage inductance of primary/secondary
\(L_{1,2m} \): mutual inductance of primary/secondary
\(L_{1,2} \): coupling inductance of primary and secondary

Note that this paper does not make use of referring variables and parameters of secondary to the primary side.

As in German Transrapid and Japanese Maglev, the long-stator of the test track, too, is divided into separately fed sections. Each section consists of series-connected stator elements and is not activated but when a shuttle or a convoy is present. When a shuttle is present the active part of the stator-section being in cooperation with the secondary of the shuttle behaves like a rotating motor. Therefore a doubly-fed linear motor can be described by the well-known equivalent circuit of slip-ring motor, see Fig. 5.

![Diagram](image)

Fig. 5 Equivalent circuit of doubly-fed long-stator motor

From the figure it can be seen that resistance and leakage inductance of the primary (long-stator) winding consists of two parts. The active part of the stator-section is covered by the secondary of a shuttle and is equal in length to the secondary. The passive part radiates its magnetic field into the air and its length depends on the length of this section. This part behaves like a series-connected inductor.

As with rotating machines main inductances are given by

\[
\begin{align*}
L_{1m} &= (N_1 \xi_1)^2 \cdot \Lambda_m \\
L_{2m} &= (N_2 \xi_2)^2 \cdot \Lambda_m \\
L_{12} &= N_1 \xi_1 N_2 \xi_2 \cdot \Lambda_m
\end{align*}
\]

(4)

\(N_{1,2} \xi_{1,2} \): effective number of turns of primary/secondary
\(\Lambda_m \): magnetic reluctance

Voltage equations can also be written like

\[
\begin{align*}
U_1 &= (R_1 + jω_1 L_{1\sigma}) I_1 + jω_1 L_{12} I_2 \\
U_2 &= (R_2 + jω_2 L_{2\sigma}) I_2 + jω_2 L_{12} I_1
\end{align*}
\]

(5)

with self-inductances

\[
\begin{align*}
L_1 &= L_{1\sigma} + L_{1m} \\
L_2 &= L_{2\sigma} + L_{2m}
\end{align*}
\]

(6)

In accordance with eq. 5 each terminal voltage consists of three parts the meaning of which for the primary are:

- \(R_1 I_1\): voltage drop at resistance,
- \(jω_1 L_{1\sigma} I_1\): voltage induced by all the magnetic field produced by the own (primary) current,
- \(jω_1 L_{12} I_2\): voltage induced by all the magnetic field produced by the current of the other (secondary) winding.

The voltage requirement of the windings \(U_{1,2} \) must not be more than the rated voltage of the power converters. It is calculated as the amounts of terminal voltages which result from

\[
\begin{align*}
U_{1}^2 &= U_{1} U_{1}^* = (R_{1}^2 + ω_1^2 L_{1\sigma}^2) I_1^2 + 2ω_1 L_{12} I_1 \cdot (ω_1 L_{12} I_2 - R_1 I_2) \\
U_{2}^2 &= U_{2} U_{2}^* = (R_{2}^2 + ω_2^2 L_{2\sigma}^2) I_2^2 + 2ω_2 L_{12} I_2 \cdot (ω_2 L_{12} I_1 + R_2 I_1)
\end{align*}
\]

(7)

3.2 Power of primary and secondary

Multiplying its voltage equation by the conjugate of the
winding's current and the number of phases derives the relation for apparent power of a winding.

\[ S_1 = 3U_1 I_1^* = 3R_1 I_1^2 + j3\omega_1 I_1^2 + 3j\omega L_{12} I_2 L_2^* \]
\[ S_2 = 3U_2 I_2^* = 3R_2 I_2^2 + j3\omega_2 I_2^2 + 3j\omega L_{12} I_1 I_2^* \]  \hspace{1cm} (8)

The amounts of apparent power determine the rated power of the power converters. They result from

\[ S_1 = 3U_1 I_1; \quad S_2 = 3U_2 I_2 = 3U_2 \sqrt{I_{2d}^2 + I_{2q}^2} \]  \hspace{1cm} (9)

and can be calculated from eq. 8 easily.

From the same equation we determine the active powers as the real parts of complex apparent powers.

\[ P_1 = 3R_1 I_1^2 + 3\omega_1 L_{12} \Re \{ I_1 I_2^* \} \]
\[ P_2 = 3R_2 I_2^2 + 3\omega_2 L_{12} \Re \{ I_1 I_2^* \} \]  \hspace{1cm} (10)

Here the left hand sides represent the power consumption of the windings while the right hand sides consist of the copper losses and the active powers withdrawn from the windings by the magnetic field.

Since the energy stored in the magnetic field is constant at steady-state operation and iron losses are neglected, the withdrawn powers equal the powers \( P_{1a}, P_{2a} \) transferred via the air gap. Simplifying and standardizing the last expression in eq. 10 and making use of \( d,q \)-components results in

\[ P_{1d} = 3\omega_1 L_{12} \Re \{ I_1 I_2^* \} = 3\omega_1 L_{12} \cdot I_1 I_{2q} \]
\[ P_{2d} = -3\omega_2 L_{12} \Re \{ I_1 I_2^* \} = -3\omega_2 L_{12} \cdot I_1 I_{2q} \]  \hspace{1cm} (11)

The sum of powers withdrawn from the windings is transformed to mechanical power for which we get

\[ P_M = P_{1d} + P_{2d} = (\omega_1 - \omega_2) L_{12} \cdot \Re \{ I_1 I_2^* \} \]  \hspace{1cm} (12)

Making use of eq. 1 results in

\[ P_M = 3\omega_M L_{12} \cdot \Re \{ I_1 I_2^* \} = -3\omega_M L_{12} \cdot I_1 I_{2q} \]  \hspace{1cm} (13)

Flow of energy is visualized by Fig. 6.

![Flow of energy in doubly-fed induction motor](image)

Note that the power consumption of the windings \( P_1, P_2 \) as well as the mechanical power \( P_M \) can be positive (solid arrow) or negative (white arrow) while the losses \( P_{1a}, P_{2a} \) can only have the given direction.

### 3.3 Thrust force

The mechanical power \( P_M = v_M \cdot F_M \) can also be expressed by mechanical quantities:

\[ P_M = 2\tau_p \cdot f_M \cdot F_M = \frac{\tau_p}{\pi} \cdot \omega_M \cdot F_M \]  \hspace{1cm} (14)

- \( F_M \): thrust force
- \( \tau_p \): pole pitch

Comparison of eq. 13 and eq. 14 results in the following expression for thrust force

\[ F_M = \frac{3\pi}{\tau_p} L_{12} \cdot \Re \{ I_1 I_2^* \} = -\frac{3\pi}{\tau_p} L_{12} \cdot I_1 I_{2q} \]  \hspace{1cm} (15)

This equation can be used to eliminate \( \Re \{ I_1 I_2^* \} \) in eq. 12 and eq. 13 and it is remarkable how simple thrust force depends on air gap power:

\[ F_M = \frac{P_M}{v_M} = \frac{P_{1d}}{v_1} = \frac{P_{2d}}{v_2} \]  \hspace{1cm} (16)

Or, when frequencies instead of speeds are used

\[ F_M = \frac{1}{2\tau_p} \cdot \frac{P_{1d}}{f_1} = \frac{1}{2\tau_p} \cdot \frac{P_{2d}}{f_2} \]  \hspace{1cm} (17)
4. Degrees of Freedom for Influencing Operation of the Drive

The behaviour of an electrical machine is determined by its currents, which are characterised by their d-components, q-components and frequencies. Since \( I_{iq} = 0 \) holds by definition (hence \( I_{id} = I_i \)) five degrees of freedom are available to control the behaviour of the motor.

- Current product \( I_i I_{2q} \) determines thrust force and is used for speed control as usual.
- Current ratio \( I_i / I_{2q} \) is important for many aspects discussed later.
- Current \( I_{2d} \) can be used to decrease voltage and power requirements of the primary winding by flux weakening. Due to the great gap flux weakening is not effective. Therefore it is not considered here and in the following we assume

\[
I_{2d} = 0
\]  \hspace{1cm} (18)

- Frequency \( \omega_2 \) determines the air gap power of the secondary. By proper choice of \( \omega_2 \) energy can be transferred to the on-board power supply as discussed in the following section.
- Frequency \( \omega_1 \) has to be chosen in accordance with the synchronism requirement (eq. 1) as \( \omega_1 = \omega_m + \omega_2 \).

5. Energy transfer to on-board power supply

If the power consumption of the secondary becomes negative \((P_2 < 0)\) energy is fed to the on-board battery of the shuttle. From eq. 11 and eq. 12 we get an equation for the power \( P_B = -P_2 \) fed to the battery

\[
P_B = -P_2 = -3R_2 I_2^2 = -3\omega_2 L_{12} \cdot I_i I_{2q} - 3R_2 I_2^2
\]  \hspace{1cm} (19)

Consequently \(-\omega_2 L_{12} I_i I_{2q} > R_2 I_2^2\) must be fulfilled for energy transfer to the battery which occurs if

\[
\frac{\omega_2 L_{12}}{R_2} \frac{I_i}{I_{2q}} > 1 \quad \text{or} \quad \frac{\omega_2 L_{12}}{R_2} \frac{I_{2q}}{I_1} > 1\]  \hspace{1cm} (20)

In accordance with this result energy transfer to the battery occurs:

- at driving thrust force \((F_M > 0, \quad I_{2q} < 0)\) if subsynchronous operation \((\omega_M < \omega_1)\) is present,
- at braking thrust force \((F_M < 0, \quad I_{2q} < 0)\) if hypersynchronous operation \((\omega_M > \omega_1)\) is present.

When losses are minimized \((I_i = I_{2q})\) eq. 20 becomes

\[
\frac{\omega_2 L_{12}}{R_2} \frac{I_i}{I_{2q}} > 1 \quad \text{or} \quad \frac{\omega_2}{R_2} \frac{I_2 q}{L_{12}} \frac{I_{2q}}{I_1} > 1
\]  \hspace{1cm} (21)

As can be seen current ratio \( I_i / I_{2q} \) determines the minimum amount of secondary frequency \( \omega_2 \) required for energy transfer to the on-board power supply.

6. Design Aspects

6.1 Determination of section length

Due to varying slope of tracks (up to 5.3% at test track) voltage required for driving the shuttle varies with position on the test track. Therefore force and current requirement was calculated for different values of slope. Under consideration of stator frequency needed for energy transfer to shuttle voltage requirement of primary was calculated as a function of section length. With the result the number of stator elements per stator sections could be chosen in such a way that only one type of power converter could be applied for all sections.

6.2 Efficiency and converter utilisation

Utilisation of active power absorbed by a drive is characterised by efficiency.

\[
\eta = \frac{P_{out}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}} = 1 - \frac{3 \cdot (R_1 I_1^2 + R_2 I_2^2)}{P_{in}}
\]  \hspace{1cm} (22)

(Note: Neglecting iron losses can cause considerable error.)

Since three active powers \((P_1, \quad P_2, \quad P_M)\) are reversible at doubly-fed motors, see Fig. 6, many cases have to be distinguished when investigating the efficiency. For operation of RailCab shuttles three cases are of special interest:

- driving the shuttle and feeding the on-board battery \( P_1 > 0; \quad P_M > 0; \quad P_B > 0; \quad P_{in} = P_1 \)
- breaking the shuttle and feeding the on-board battery \( P_1 > 0; \quad P_M < 0; \quad P_B > 0; \quad P_{in} = P_1 - P_M \)
• accelerating the shuttle with assistance of battery
\[ P_B > 0; \quad P_M > 0; \quad P_B < 0; \quad P_H = P_1 - P_B \]

It is not surprising that efficiency, too, is influenced strongly by current ratio \( I_1/I_2 \). As can be shown easily by evaluating eq. (22) efficiency becomes optimum when the current ratio matches \((I_1/I_2)^2 = R_2/R_1\), i.e. when losses of primary and secondary are equal. According to this requirement stator sections should be approximately as long as the secondary. Normally this optimisation resulting in very short stator sections will not be realised because a great number of (small) power converters would be required.

Overall size of motors and power converters depend on apparent power. To characterise size of and expense for these units a new characteristic called converter utilisation is suggested at which active output power of the system is related to apparent power transmitted by the converters:

\[
\eta_{SC} = \frac{P_{out}}{S_1 + S_2} = \frac{P_H - 3 \cdot (R_1 I_1^2 + R_2 I_2^2)}{S_1 + S_2} \quad (23)
\]

Like efficiency this quantity is also determined by current ratio \( I_1/I_2 \). When calculated for operation of drive with and without energy transfer the difference characterises the additional expense for contact-less energy transfer. For a full-scale system consideration of this difference and expense for overhead wires or contact rails will determine whether power supply of shuttles via the motor is economically advantageous or not.

6.3 Normal force of linear motor

A special problem of linear motors is normal force which stresses rails and sleepers on the trackside as well as the whole undercarriage of the shuttle. It is determined by the magnetic flux density of the air gap and depends on the current ratio \( I_1/I_2 \), too, as will be shown in the following.

Primary and secondary current contribute in exciting the field in the air gap and their contributions are

\[
B_1 = \frac{3 \mu_0 N_1 \xi_1}{\pi \delta''} \cdot \sqrt{2} I_1 \cdot \sin(\pi \frac{x}{\tau_P})
\]
\[
B_2 = \frac{3 \mu_0 N_2 \xi_2}{\pi \delta''} \cdot \sqrt{2} I_2 \cdot \sin(\pi \frac{x}{\tau_P} + \alpha)
\]

Calculation of normal force \( F_z \) starts from the basic formula

\[
\frac{dF_z}{dA} = -\frac{1}{2 \mu_0} \cdot B_2^2 = -\frac{1}{2 \mu_0} \cdot (B_1 + B_2)^2
\]

\[(A: \text{ air gap area}) \quad \text{and, with eq. 24, results in}
\]

\[
F_z = \frac{9 \mu_0 b_p \tau_p}{\pi^2 \rho \delta''^2} \cdot \left((N_1 \xi_1 I_1)^2 + N_2 \xi_2 I_2)^2 + 2 \cdot N_1 \xi_1 I_1 \cdot N_2 \xi_2 I_2 \cdot \cos \alpha\right)
\]

\( \mu_0: \) permeability of air
\( b_p: \) width of linear motor core
\( \rho: \) number of pole pairs
\( \delta'': \) effective air gap (distance from iron to iron)

With the expression for coupling inductance

\[
L_{12} = \frac{6 \mu_0 \tau_p b_p p}{\pi^2 \rho \delta''} \cdot N_1 \xi_1 I_1 \cdot N_2 \xi_2 I_2
\]

normal force, given at eq. (15), can also be expressed in terms of motor design parameters

\[
F_M = \frac{18 \mu_0 b_p}{\pi \rho \delta''} \cdot N_1 \xi_1 I_1 \cdot N_2 \xi_2 I_2 \cdot \sin \alpha
\]

and finally we get for the ratio of normal and thrust force

\[
\frac{F_z}{F_M} = \frac{\tau_p}{2 \pi \delta''} \cdot \frac{N_1 \xi_1 I_1 + N_2 \xi_2 I_2 + 2 \cdot \cos \alpha}{\sin \alpha}
\]

Concluding this investigation we can establish that, with given thrust force normal force becomes small

• when the air gap is great related to pole pitch.
• when \( \alpha = \pm 90^\circ \) holds i.e. when force is generated with optimum displacement of primary and secondary field

\[
\frac{F_z}{F_M} = \frac{\tau_p}{2 \pi \delta''} \left(\frac{N_1 \xi_1 I_1 + N_2 \xi_2 I_2}{N_1 \xi_1 I_1 + N_2 \xi_2 I_2}\right)
\]

• when \( N_1 \xi_1 I_1 = N_2 \xi_2 I_2 \) is realised i.e. when contributions of primary and secondary to air gap field are equal. In this case we simply get
\[
\frac{F_x}{F_M} = \frac{\tau_p}{\pi S''}
\]  

(31)

The condition for minimum normal force becomes coherent, when looking at the phasors of currents as shown at Fig. 7. For minimum copper losses current phasors \( I_1 \) of primary and \( I_2^{(1)} \) of secondary are kept at right angles. (\( I_2^{(1)} \): secondary current referred to primary). For constant force the product of their magnitudes must be constant. Therefore, the locus of the magnetizing current \( I_m = I_1 + I_2^{(1)} \) is a hyperbola at which \( |I_1| \cdot |I_2^{(1)}| = \text{const} \).

![Diagram of current phasors](image)

Fig. 7 Diagram of current phasors

From the figure can be seen, that the magnitude of the magnetizing current (\( I_{m,\text{opt}} \)) and consequently the total field in the air gap and the normal force become minimum when \( |I_1| = |I_2^{(1)}| \) i.e. when contributions of primary and secondary to air gap field are equal.

For long-stator linear motor of test track force ratio was calculated as \( F_y/F_x = 3 \) while measurement delivered \( F_y/F_x = 2.7 \) (note: with comparable short-stator induction motors ratio is expected \( F_y/F_x > 7 \)).

7. Conclusion

In accordance with the requirements of the shuttle-based RailCab system a doubly-fed linear motor was chosen which allows operation of shuttles at different speeds and transfer of energy to the shuttles without overhead wires or contact rails. When designing the motor the ratio of primary and secondary currents proves to be an important parameter which determines the efficiency of the drive, the rated power of the track-side and shuttle-side inverters as well as the normal force of the motor. Making use of energy transfer via the linear motor demands for bigger motor parts and inverters. After utilisation of motor for energy transfer has already been tested successfully at the test track evaluation of economic efficiency will determine if it is suitable for full size application.

References


Horst Grotstollen was born in Mannheim, Germany, in 1938. He received Ing.-grad. and Dipl.-Ing. and Ph.D. degree in electrical engineering from Staatliche Ingenieurshule Duisburg in 1960, from Rheinisch-Westfälische Technische Hochschule Aachen in 1965 and from Technische Universität Berlin in 1972, respectively. He habilitated at Universität Erlangen-Nuernberg in 1982. He joined AEG from 1965 through 1973 developing electrical servo drives in the Frankfurt Research Center and dealing with drive applications in the Department of Industrial Equipment. After 1973 he was Chief Engineer at the Chair of Electrical Drives of the University of Erlangen-Nuernberg where he habilitated in 1982 with a thesis on Introduction of Three-Phase Technology to Servo Drives. From 1981 through his retirement in 2003 he was professor at University of Paderborn doing research in field of digital control of ac drives, switch mode power supplies and their magnetic components and control and supply of piezoelectric devices. He is member of VDE, IEEE and RailCab consortium.