An Introduction to Resonant Operated Piezoelectric Actuators

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1 Introduction

If a force is applied to an element of piezoelectric ceramics an electric field E will appear in the x_3-axis in which the element is polarized. Consequently a voltage u_P can be measured between electrodes which are mounted to the surfaces and electric charge will be present at the electrodes which means that mechanical energy has been converted to electrical energy due to the piezoelectric effect. For a long time this phenomenon has been used at sensors for measuring forces, pressure and quantities which cause forces like acceleration. The piezoelectric effect is reversible and can be used for electromechanical energy conversion, too.

During the last years piezoelectric actuators are becoming more and more attractive due to their high power density in particular when operated at high frequency and when making use of resonant amplification in the mechanical part. But not only electromechanical energy conversion is possible: piezoelectric transformers can be easily realized by simply implementing two pairs of electrodes on the same piece of piezoelectric ceramics. Up to now piezoelectric actuators have been realized with low power only (motors less than 100 W, transformers less than 10 W), but high efforts are made to increase power ratings. This is why in education on electrical drives some attention should be spent to this field. This contribution will show what an introductory lecture on piezoelectric actuators could be like and what knowledge on conventional electrical systems can be applied for better understanding of the mechanical systems. For general information see [1], [2].

2 Basic equations of electromechanical energy conversion

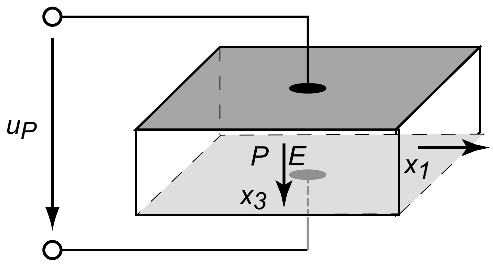

Fig. 1: Basic piezoelectric actuator

If a voltage u_P is applied to the electrodes of a piezoelectric element an electric field E is applied to the x_3-axis in which it is polarized and a deformation occurs in the x_3-axis (d33-effect) as well as in the quadrature x_1-axis (d31-effect), see Fig. 1. From the permitted electric field strength of 2000 V/mm results the maximum change of dimensions which is less than 0,2 %. Therefore the basic piezoelectric actuator is well suited to achieve high forces at a small stroke but only a very small amount of energy is converted at each stroke.

The small amount of energy dw_i which is converted electromechanically when any actuator performs a very small movement of dx can be expressed by electrical or mechanical quantities as

$$dw_i = f_i \cdot dx = u_i \cdot i_i \cdot dt \quad . \tag{1}$$

where f_i, u_i and i_i are an inner force, an inner voltage and an inner current, respectively. By introducing the speed $v = dx/dt$ we get a similar expression for the inner power $p_i = dw_i/dt$ of the actuator

$$p_i = f_i \cdot v = u_i \cdot i_i \quad . \tag{2}$$

From this equation we can derive two expressions for the two types of actuators

$$\text{a)}\ \frac{f_{Pi}}{u_P} = \frac{i_{Pi}}{v} = A_P \qquad\qquad \text{b)}\ \frac{f_{Mi}}{i_M} = \frac{u_{Mi}}{v} = K_M \tag{3}$$

where A_P is a constant in case of piezoelectric actuators while K_M is a constant in case of magnetic actuators e.g. a permanent exactitude dc motor. Thus, piezoelectric and magnetic actuators are reciprocal systems at which for instance the force depends either on an inner voltage or on an inner current: According to eq. 3 the inner mechanical and electrical quantities of a) piezoelectric and b) magnetic actuators are related by

$$\text{a)}\ f_{Pi} = A_p \cdot u_P; \quad i_{Pi} = A_P \cdot v \qquad\qquad \text{b)}\ f_{Mi} = K_M \cdot v_P; \quad u_{Mi} = K_M \cdot v \tag{4}$$

3 Piezoelectric actuators at quasi static operating conditions

The inner quantities differ from the quantities which are applied to the terminals because energy is stored in the electric field of a piezoelectric actuator and in the magnetic field of a magnetic actuator's windings. Furthermore losses have to be considered which mainly depend on voltage at piezoelectric actuators, see Fig. 2, and on current at magnetic actuators. For the terminals quantities of a) piezoelectric and b) magnetic actuator we get

$$\text{a)}\quad i_P = C_P \cdot \frac{du_P}{dt} + \frac{1}{R_P} \cdot u_P + i_{Pi}$$
$$\text{b)}\quad u_M = L_M \cdot \frac{di_M}{dt} + R_M \cdot i_M + u_{Mi} \tag{5}$$

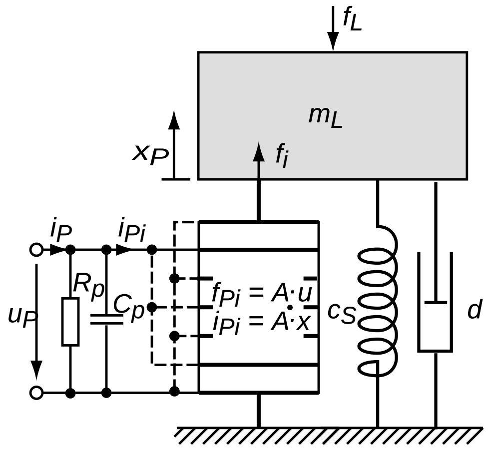

Fig. 2: Piezoelectric actuator loaded by prestress spring and load mass

On the other side at both actuators the generated force f_i differs from the external force f_L of the load. At magnetic actuators mainly the force required for acceleration of mass m_L and friction has to be considered. At piezoelectric actuators situation is much more complex and depends on operation frequency.

At low operation frequency (quasi static conditions) a lumped mass m_L can be assumed but the stiffness c_S of a spring must be considered which is used to prestress the piezoceramic material to avoid tensile strain, see Fig. 2. Thus, for the mechanical system of a) piezoelectric and b) magnetic actuators we get

$$\text{a)}\quad m_L \cdot \frac{d^2 x_P}{dt^2} = f_{Pi} - f_L - d_L \cdot \frac{dx_P}{dt} - c_S \cdot x_P \qquad \text{b)}\quad m_L \cdot \frac{d^2 x_M}{dt^2} = f_{Mi} - f_L - d_L \cdot \frac{dx_M}{dt} \tag{6}$$

The equation systems consisting of eq. 4, eq. 5 and eq. 6 can be represented by block diagrams or equivalent circuits. For piezoelectric actuators and systems the block diagram is shown at Fig. 3.

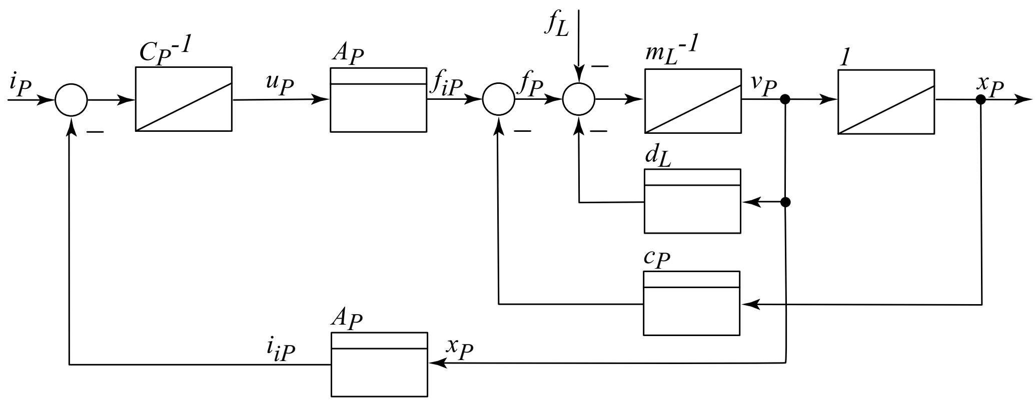

Fig. 3: Block diagram of piezoelectric actuator loaded by prestress spring and load mass

The equation of the equivalent circuit is derived when eq. 4 is used to substitute the mechanical quantities in eq. 6 by mechanical quantities. By this measure we get the differential equation of piezoelectric actuators

$$\frac{m_L}{A_P^2} \cdot \frac{di_{Pi}}{dt} + \frac{d_L}{A_P^2} \cdot i_{Pi} + \frac{c_s}{A_P^2} \cdot \int i_{Pi}\ dt \ = \ u_P - \frac{1}{A_P} \cdot f_L\ , \tag{7}$$

which is equivalent with the equation of a series resonant circuit

$$L_M \cdot \frac{di_{Pi}}{dt} + R_M \cdot i_{Pi} + C_M \cdot \int i_{Pi}\ dt \ = \ u_P - u_L \tag{8}$$

the parameters and variables of which are

$$L_M = \frac{1}{A_P^2} \cdot m_L; \quad R_M = \frac{1}{A_P^2} \cdot d_L; \quad C_M = \frac{1}{A_P^2} \cdot c_S$$

$$u_x = \frac{1}{A_P} \cdot f_x; \qquad i_{Pi} = A_P \cdot v \tag{9}$$

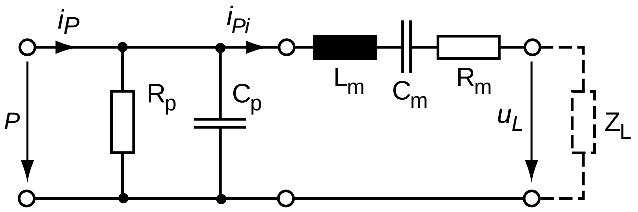

Fig. 4: Equivalent circuit of loaded piezoelectric actuato

The equivalent circuit including the electrical part is shown at Fig 4. Note that the inner current i_{Pi} represents the speed of the actuator and the load while the voltages at the inductance L_M, the capacitance C_M and the resistance R_M represent the forces accelerating the mass, stressing the string and causing losses in the mechanical part.

Again piezoelectric and magnetic systems and prove to be reciprocal as can be seen from Table 1 where the electrical representatives of mechanical parameters and quantities are summarized.

Table 1: Electric equivalents of mechanical quantities

Mechanical quantity	represented by electrical quantity	
	piezoelectric system	magnetic system
force	voltage	current
speed	current	voltage
losses	resistor	resistor
mass	inductance	capacitance
elasticity	capacitance	inductance

4 Piezoelectric actuators at transient operating conditions

Because of the small deformation of the ceramics only a small amount of energy is converted at each stroke. Therefore most piezoelectric systems are operated at high frequencies in the ultrasonic range to increase the converted power. Due to high frequencies the elasticity and mass of the ceramics cannot be neglected; even more, these are essential for the resonant operation of the actuators which make use of structural oscillations.

The mechanism of structural oscillations is explained at a passive bar in which a plain wave is induced by a thin piezoelectric actuator at $x = 0$. The waves propagate in x-axis, see Fig. 5. To derive the wave equation a thin slice of thickness x_Δ and mass m_Δ is considered. Calculation of its deviation ξ from the initial position x and its compression dx_Δ and strain σ results in the wave equations

$$\frac{d^2\xi}{dt^2} = v_S^2 \cdot \frac{d^2\xi}{dx^2} \qquad \text{and} \qquad \sigma = -E \cdot \frac{d\xi}{dx} \tag{10}$$

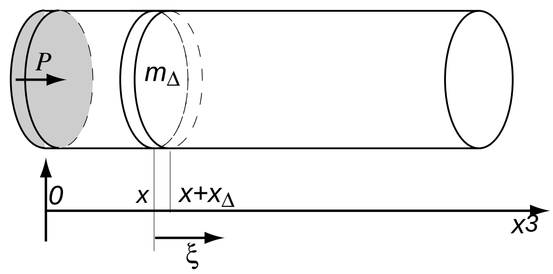

Fig. 5: Definition of coordinates

where the velocity of sound $v_S = \sqrt{E/\rho}$ is determined by the modulus of elasticity E and the density ρ of the material. Velocity of sound is 5,200 m/s for steel and 2,500 .. 4,600 m/s for piezoelectric ceramics.

Solving the wave equation results in

$$\xi(x,t) = (A \cdot \cos(2\pi f \cdot t) + B \cdot \sin(2\pi f \cdot t)) \cdot \left(C \cdot \cos\left(2\pi \cdot \frac{x}{\lambda}\right) + D \cdot \sin\left(2\pi \cdot \frac{x}{\lambda}\right)\right)$$
$$\sigma(x,t) = -\frac{2\pi \cdot E}{\lambda} \cdot (A \cdot \cos(2\pi f \cdot t) + B \cdot \sin(2\pi f \cdot t)) \cdot \left(D \cdot \cos\left(2\pi \cdot \frac{x}{\lambda}\right) - C \cdot \sin\left(2\pi \cdot \frac{x}{\lambda}\right)\right) \tag{11}$$

where f and λ represent the frequency and the wave length, respectively, which are related by $\lambda = v_S/f$. Thus, at a frequency of 20 kHz wave length is appro.. 260 mm for steel and 125 ... 230 mm for piezoelectric ceramic material.

The constants of integration have to be calculated from the boundary conditions. In case of the bar shown at Fig. 5 these are $\xi(x = 0, t) = 0$ and $\sigma(x = L, t) = 0$ delivering $C = 0$ and $\cos(2\pi \cdot L/\lambda) = 0$. The last equation is satisfied for an infinite number of discrete resonance frequencies satisfying

$$\lambda_k = \frac{1}{2k+1} \cdot 4L \qquad f_k = \frac{v_S}{\lambda} = (2k+1) \cdot \frac{v_S}{4L} \qquad \text{mit} \quad k = 1, 2, 3, ... \tag{12}$$

An example of practical importance is given at Fig. 6. The bar shown here is excited and fixed at $x = L/4$ (therefore $\xi(L/4, t) = 0$) with a frequency for which $\lambda = L$ holds. At both surfaces of the bar no force and no strain is present ($\sigma(0, t) = 0 = (L, t)$). Due to these boundary conditions resonance will appear for

$$L/4 = k \cdot \lambda_k/4 \quad \lambda_k = L/k \quad f_k = k \cdot v_S/L \ . \tag{13}$$

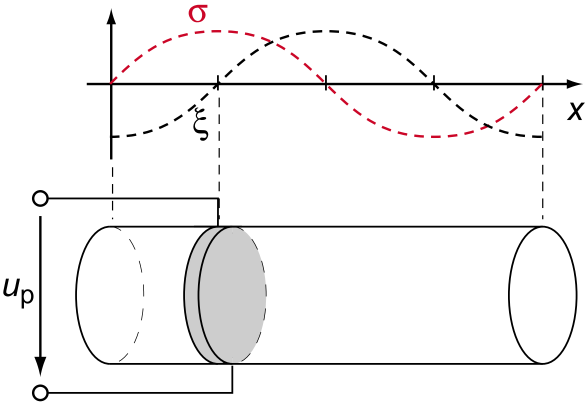

Fig. 6: Resonant operated sonotrode

If eq. 13 is satisfied no deviation appears at $x = L/4$ and therefore the support of the bar is not stressed. But strain and compression are maximum at this position.

For transfer of power from the actuator to a load that is coupled to the bar's surface strain as well as deviation must be present at the positions of the actuator ($x = L/4$) and of the surface (e.g. at $\lambda = L$). These requirements can be fulfilled by a variation of frequency and wave length which is established automatically by a suitable control.

Furthermore the deviation appearing at $x = L/2$ can be increased by reducing the bar's cross-sectional area. In Fig. 7 reduction is applied at $x = 3L/4$ where no deviation takes place. Since the longitudinal force f must be equal on both sides of the cross-sectional reduction

$$\frac{\hat{\sigma}_2}{\hat{\sigma}_1} = \frac{A_1}{A_2} \qquad \text{and} \qquad \frac{\hat{\varepsilon}_2}{\hat{\varepsilon}_1} = \frac{\hat{\xi}_2}{\hat{\xi}_1} = \frac{A_1}{A_2} \tag{14}$$

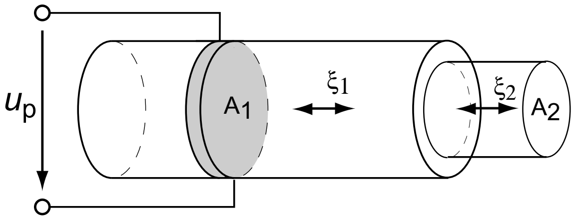

Fig. 7: Sonotrode with cross-sectional reduction

must hold for the amplitudes of strain $\hat{\sigma}$, compression $\hat{\varepsilon}$ and deviation $\hat{\xi}$. Note that in practice the cross-sectional reduction must not be introduced as abrupt as in Fig. 7 to avoid destruction by notch effect.

Considering the equations and figures of this section a close relation can be established between wave propagation at piezoelectric systems and electrical transmission lines. That is why all results achieved from transmission equations can be applied to piezoelectric systems and, for example, the parameters of a four-pole network can be calculated from the dimensions of the mechanical system and the operating frequency. The result is an equivalent circuit as shown at Fig. 4, but having an infinite number of resonant circuits in parallel because resonant oscillation can appears at all frequencies satisfying eq. 13. In practice a piezoelectric system will be operated in the close surrounding of one resonance frequency. Therefore, normally only one mechanical resonant circuit has to be considered in the equivalent circuit which in this case is identical to Fig. 4.

5 Design and applications

In practice two basic types of piezoelectric actuator systems (sonotrodes) have to be distinguished:

Sonotrodes as considered up to now are normally equipped with stack actuators consisting of thin ceramic slices which are alternatingly polarized and are separated by electrodes according to Fig. 8. By this measure very high voltages can be avoided which would be required to activate monolitic actuators of great thickness. For the design of stack actuators odd numbers of slices are pre-

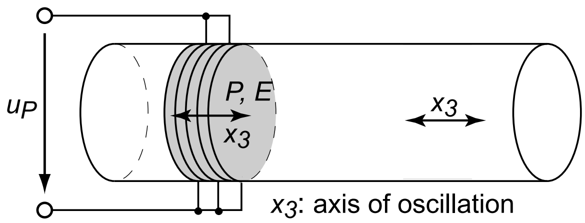

Fig. 8: Sonotrode with stack actuator making use of d_{33}-effect

ferred because in this case both electrodes being in contact with the oscillating steel bar can be grounded. This kind of actuator can be used at machine tools to improve the performance of machining processes by superimposing an ultrasonic oscillation in quadrature to the cutting speed.

A piezoelectric transformer using d_{33}-effect comes into being when a second actuator is introduced into the system as shown at Fig. 9a). The transmission ratio of the transformer is determined by the design of the piezoelectric actuators. The system is operated at $\lambda = L$ as shown by the dotted lines of Fig. 9b). Excellent isolation of primary and secondary is achieved if the whole bar is made from ceramics and no steel is used.

The actuator in the left half of Fig. 9c) makes use of the d_{31}-effect. Polarisation and electrical field which determine x_3 -axis are oriented in the vertical direction while the oscillating bar is excited by the deformation in the horizontal x_1 -axis. Again the whole oscillating system consist of one piece of piezoceramic. The principle of this actuator is used at a piezoelectric transformers proposed by Rosen in 1958 [3] at which the right half of the ceramic element is polarized in the horizontal axis. If secondary electrodes are placed on the right surface and in the plane dividing the left and the right half than high voltages can be achieved. Since inserting an electrode in the middle of the ceramics causes problems one of the primary electrodes is normally used for the secondary, too, and the unit behaves like an autotransformer.

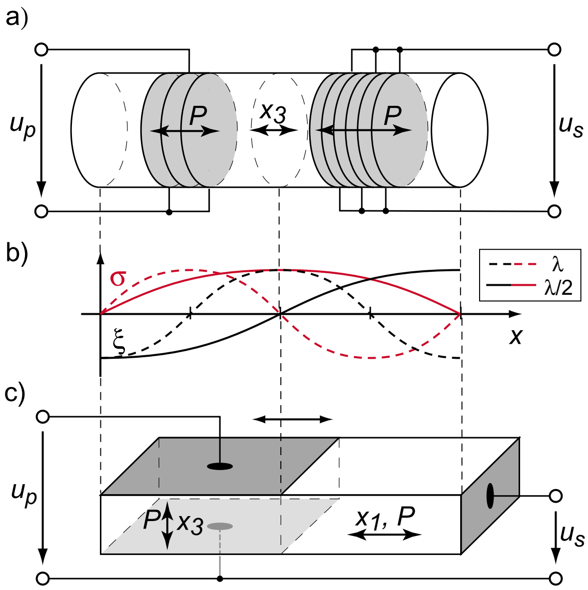

Fig. 9: Piezoelectric transformers
a) using d_{33}-effect
c) using d_{31}-effect
b) distribution of strain and deviation

Piezotransformers can also be designed by using ring-shaped ceramics. They offer small size and good efficiency and have been realized for up to 10 W in the power supply for a handy [4].

For drive application the oscillation of the piezoelectric actuator has to be transferred to a continuous motion for which different proposals have been realized. At ultrasonic travelling wave motors a travelling wave is excited on the circumference of a brass disc [5]. Travelling wave motors with rated power up to 100 W generate high torque at low speed and are in use at autofocus cameras and for steering wheel adjustment in cars [6].

Last not least an aspect concerning power supply and control of piezoelectric actuator should be mentioned: Due to their input capacitance piezoelectric actuators need to be decoupled from voltage source inverters by a series inductor which can be used for compensation of reactive power. In connection with the actuator's capacitance this inductor forms a second resonant circuit and attention has to be payed to avoid undue electrical or mechanical strain by a suitable control. Design of the control is not trivial because the oscillation of two resonant systems having varying parameters and being operated in their resonance have to be controlled [7].

Literature:

[1] Philips: Piezoelectric Ceramics - Properties and Applications. Philips Gloeilampenfabrieken, Eindhoven, Niederlande, 1991.

[2] Taylor, G. W. et al.: Piecoelectricity, Ferroelectricity and Related Phenomena. Vol. 4, Gordon and Breach Sciences Publishers, 1985.

[3] Rosen, Charles A. et al.: Electromechanical Transducer. US-Patent 2,830,274, 1958.

[4] Navas, J. et al.: Miniaturised Battery Charger using Piezoelectric Transformers. 16th Annual IEEE Applied Power Electronics Conference (APEC 2001), Anaheim, California, USA, Vol.1, pp. 492-496.

[5] Sashida, T.; Kenjo, T.: An Introduction to Ultrasonic Motors. Clarendon Press, Oxford, 1993.

[6] Wallaschek. J.: Piezoelectric Ultrasonic Motors. Journal of Intelligent Materials and Structures (1995), Vol. 6, Nr. 1, pp. 71-83.

[7] Maas, J.et al.: High Performance Speed Control for Inverter-fed Ultrasonic Motor Optimized by a Neural Network. 6th Int. Conf. on New Actuators. ACTUATOR'98, Bremen, 1998, pp. 260-263.