ANALYSIS OF A SERIES–PARALLEL-LOADED RESONANT-CONVERTER WITH ‘TERTIARY SIDE RESONANCE’ FOR HIGH OUTPUT POWER AND LOW OUTPUT VOLTAGE

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Abstract: A series–parallel–loaded resonant–converter operating above resonance is proposed where the parallel tank capacitor is connected to the resonant circuit through a tertiary transformer winding. Thereby a part of the leakage inductance of the transformer is utilized as resonant element and by proper choice of the transformer ratio the ac–current in the parallel tank capacitor and the capacitor voltage is reduced. General solutions for the state variables, converter gain, and component stress are obtained by an analysis in the time domain for steady-state operation including effects of the high-frequency transformer.

Keywords: Resonant–Converter, Component Stress, Operation above Resonance

INTRODUCTION

A common way to reduce the size of switch–mode power–supplies and to improve the transient response is to increase the switching frequency $f_s$. Because of the increasing switching losses with raised $f_s$ in today used PWM–converters resonant converters have become interesting again. It has been shown by Steigerwald (1988) and Kunze (1989) that the series–parallel–loaded resonant–converter, known as SPRC or LCC-type parallel converter, operating above resonant frequency $f_r$, i.e. $f_s > f_r$, has many desirable features such as lossless switching with capacitive snubbers leading to excellent efficiency characteristics and low frequency range with wide input voltage and load range. For low output voltage and high output power applications it is not possible to utilize the leakage inductance of the transformer as resonant element by placing the parallel tank capacitor $C_P$ on the secondary side of the transformer because $C_P$ would have to carry an ac-current too high. With the converter proposed in this paper it is possible to overcome these drawbacks. Here $C_P$ is connected to the resonant circuit through a tertiary transformer winding – that’s why the converter is called “SPRC with tertiary side resonance” (SPRCT).

By proper choice of the primary to tertiary transformer ratio $n_3$ the tertiary ac-current is reduced in order to use capacitors for higher voltages with ordinary rms-current ratings. Thus a technological solution results which is free of costly and high stressed components and makes use of parasitics, i.e. the leakage inductances of the transformer.

OPERATION OF A SPRCT ABOVE RESONANCE AND MODELLING

Figure 1 shows the circuit diagram of a half-bridge series–parallel–loaded resonant–converter with tertiary–side resonance. The switches $S_1$ and $S_2$, consisting of transistor $T_1$ ($T_2$) and antiparallel diode $D_1$ ($D_2$) (discrete or parasitic as for MOSFETs), operate in push–pull action. The capacitors $C_1$ and $C_2$ form the series–resonant capacitor of value $C_R = C_1 + C_2 = 2C_1$, the inductor $L_r$ together with the primary and tertiary transformer leakage inductances $L_{p1}$ and $L_{p3}$ the resonant inductor. The voltage across the secondary transformer winding is rectified by $D_3$ to $D_4$ and smoothed by a L-C output filter ($L_f$, $C_f$). It has been found that the converter operates in two different modes. Referring to figures 1, 2 and 3a operation in Mode 1 can be described as follows. The switching period starts with the turn–off of transistor $T_1$. The voltage across $C_R$ is positive and diodes $D_3$ and $D_6$ are conducting. The inductor current $i_L$ changes from $S_1$ to $S_2$ (first to $D_3$ and at zero-crossing to $T_2$). $L_1$ and $L_2$ resonate with $C_f$ and $C_S$. When the input voltage of the bridge rectifier, i.e. the inverter output voltage, $u_{12}$, reaches zero output current $i_L$ commutates from $D_3$, $D_4$ to $D_1$, $D_5$. $u_{12}$ is clamped to zero during the commutation interval until $i_L$, the current in the secondary transformer winding, has reversed from $+i_L$ to $-i_L$. A resonant interval similar has the first one follows where the output current sink has changed its direction, see figure 2c. At half of the switching period, i.e. at $t = T_s/2$, transistor $T_2$ is turned–off and in the second half of the switching period the operation described above repeats with $S_1$ conducting.

For lighter loads and higher switching frequencies the converter enters into Mode 2. In this case the transistor is turned–off while the commutation of the output current has not finished. Therefore the commutation interval is divided into two intervals. A switching period starts with a commutation interval, followed by the energy–transfer interval and then by a commutation interval as can be seen in figure 3c.

ANALYSIS

Assumptions

The analysis has been performed with the following assumptions:

- The switches, diodes and resonant elements are supposed to be ideal.
Fig. 1 Circuit diagram of a half-bridge series-parallel-loaded resonant-converter with tertiary side resonance

\[ L_1 = L_r + L_{s1} \quad L_3 = L_{s3} \] (1)

Fig. 2 Equivalent circuit diagrams of the SPRCT
(Active branches marked by bold face lines)

- a) energy-return/transfer interval \([u\omega t_o, u\omega t_1]\)
- b) commutation-interval \([u\omega t_1, u\omega t_2]\)
- c) energy-transfer interval \([u\omega t_2, u\omega t_3]\)

- Input filter capacitor and output filter inductor are large enough to assume constant input voltage \(U_i\) and ripplefree output current \(I_o\). The output filter together with the load can be treated as a constant-current sink.

- The effects of the snubber capacitors across the switches S1 and S2 are neglected.

- The three-winding-transformer is represented by a 'T'-equivalent circuit (presented e.g. by Middlebrook and Cuk (1981)) where the primary and tertiary leakage inductances are considered. The secondary leakage inductance cannot be utilized as a resonant element and must be minimized at transformer realization. Therefore it is assumed to be zero. Fourth order differential equations would result, if \(L_{s3}\) is taken into account.

General solutions

The analysis reveals that a switching period can be divided into six intervals with three intervals being sufficient to describe the behaviour of the converter mathematically because of symmetry between first and second halfperiod.

Node and mesh analysis of the equivalent circuits shown in figure 2 for the three intervals leads to the following equations for the four state variables \(i_L\), current in the resonant inductor, \(i_S\) current in the tertiary winding, \(u_{CS}\) voltage at the series-resonant capacitor and \(u_{CP}\) voltage at the parallel-resonant capacitor. Because of the neglected secondary leakage inductance \((L_2 = 0)\) only differential equations of maximum order two appear.

Energy-return/transfer interval:

\[
\frac{d^2}{dt^2} i_L(t) + \frac{C_S + C_P}{C_S \cdot C_P} \frac{1}{L_3} i_L(t) = \frac{1}{C_P L_{s3}} I_a \quad \text{(2)}
\]

\[
i_S(t) = i_S(t) - I_a \quad \text{(3)}
\]

\[
u_{CP}(t) = \frac{1}{C_P} \int i_S(r) \, dr \quad \text{(4)}
\]

\[
u_{CS}(t) = \frac{1}{C_S} \int i_L(r) \, dr \quad \text{(5)}
\]

Commutation-interval:

\[
\frac{d^2}{dt^2} i_L(t) + \frac{1}{L_3 C_S} i_L(t) = 0 \quad \text{(6)}
\]

\[
\frac{d^2}{dt^2} i_S(t) + \frac{1}{L_3 C_P} i_S(t) = 0 \quad \text{(7)}
\]

\[
u_{CP}(t) = \frac{1}{C_P} \int i_S(r) \, dr \quad \text{(8)}
\]

\[
u_{CS}(t) = \frac{1}{C_S} \int i_L(r) \, dr \quad \text{(9)}
\]

Energy-transfer interval:

\[
\frac{d^2}{dt^2} i_L(t) + \frac{C_S + C_P}{C_S \cdot C_P} \frac{1}{L_3} i_L(t) = -\frac{1}{C_P L_{s3}} I_a \quad \text{(10)}
\]

\[
i_S(t) = i_S(t) + I_a \quad \text{(11)}
\]

\[
u_{CP}(t) = \frac{1}{C_P} \int i_S(r) \, dr \quad \text{(12)}
\]

\[
u_{CS}(t) = \frac{1}{C_S} \int i_L(r) \, dr \quad \text{(13)}
\]

Solving these equations leads to nonlinear equations for the state variables for each interval, containing the initial values \(i_L(\theta\cdot), i_S(\theta\cdot), u_{CS}(\theta\cdot), u_{CP}(\theta\cdot)\), the interval durations \(\theta\cdot\) (with \(i = 0, 1, 2\)), and the load current as unknowns. For steady-state the values of the state variables at the end of the third interval, i.e. at \(t = t_3 = T_S/2\), can be expressed by the initial values of the first interval using the condition that they are of the same amount but of different sign (because of symmetry between first and second halfperiod).

The average output voltage, important for calculating converter gain, can be derived from the rectified inverter output voltage \(u_{12}(t)\), i.e. \(U_a = u_{12}(t) = |u_{12}(t)|\). All equations are normalized using the following base quantities:

\[ U_B = U_e \quad I_B = \frac{U_B}{Z_B} \quad \text{with} \quad Z_B = \sqrt{L_3 + L_2} \div C_S = \sqrt{L_{s3}} \div C_S \]

An equation system with thirteen equations for Mode 1 and fourteen equations for Mode 2 follows which was solved numerically.
Fig. 3 Typical calculated waveforms of a SPRCT for

a) Mode 1: $a = 1.0$, $\beta = 0.9$, $Q = 2.5$, $f_n = 1.307$, $M = 0.31$

b) Boundary between Mode 1 and Mode 2
   $a = 1.0$, $\beta = 0.9$, $Q = 0.6$, $f_n = 1.725$, $M = 0.282$

c) Mode 2: $a = 1.0$, $\beta = 0.8$, $Q = 1.5$, $f_n = 1.813$, $M = 0.097$
The equation system for Mode 1 is given below as an example where the subscript 'n' marks normalized values and the subscript '0', '1' or '2' the border of an interval looked at.

\[ u_{cpn0} = -i_{Ln0}D + QMAD \cdot \sin(03) + QM \cdot 03 \cdot (AD - D) \]

\[ u_{cpn2} = i_{Ln2} - QM \cdot \frac{1}{H} \cdot \sin(\gamma 92) + u_{cpn3} \cdot \cos(\gamma 92) \]

\[ u_{cpn1} = (i_{Ln0} - QMAD) \cdot \sin(01) + QM \cdot (AD - D) + u_{cpn0} + A \cdot (u_{cn2n} + u_{cpn1}) \cdot \cos(01) - 1 \]

\[ i_{Ln0} = i_{Ln} \cdot \cos(03) + (u_{cn2n} + u_{cpn1}) \cdot E \cdot \sin(03) + QMA[1 - \cos(03)] \]

\[ i_{Ln2} = i_{Ln} \cdot \cos(\gamma 92) - u_{cn2n} \cdot G \cdot \sin(\gamma 92) \]

\[ i_{Ln1} = i_{Ln0} \cdot \cos(01) - (u_{cn2n} + u_{cpn0}) \cdot E \cdot \sin(01) + QMA[1 - \cos(01)] \]

\[ QM = -i_{Ln} \cdot \cos(\gamma 92) - u_{cn2n} \cdot G \cdot \sin(\gamma 92) + (i_{Ln1} - QM) \cdot \cos(\gamma 92) - u_{cpn1} \cdot H \cdot \sin(\gamma 92) \]

\[ 0 = [QM \cdot (DC - AD) + i_{Ln0} \cdot (DF - F)] \cdot \sin(01) + QM \cdot 01 \cdot (AD - D) + u_{cpn0} + (u_{cn2n} + u_{cpn0}) \cdot \cos(01) \cdot (A - C) - A \]

\[ u_{cn2n} = G = \frac{B}{E} \cdot \sin(01) + B \cdot (u_{cn2n} + u_{cpn0}) \cdot \cos(01) - QMBD \cdot \sin(01) \]

\[ u_{cn2n} = \left[ QM(AD - DC - BD) + i_{Ln0} \cdot (DF - F) \right] \cdot \sin(03) + u_{cpn2} \cdot \cos(03) \cdot (A - C - B - A) + u_{cpn3} - QM \cdot 03 \cdot (AD - D) + 1 \]

\[ M \cdot A = \frac{\theta}{2} = \frac{QM \cdot (CD - AD) + i_{Ln0} \cdot (DF - F) \cdot [1 - \cos(01)]}{2} + \frac{QM \cdot 01}{2} \cdot (AD - D) + u_{cpn0} \cdot 01 + (u_{cpn2} + u_{cpn3}) \cdot \sin(01) \cdot (A - C) - A \cdot 01 + [QM \cdot (CD - AD) + i_{Ln0} \cdot (DF - F)] \cdot [1 - \cos(03)] + \frac{QM \cdot 03}{2} \cdot (AD - D) - u_{cpn2} \cdot 03 - (u_{cpn2} + u_{cpn3}) \cdot \sin(03) \cdot (A - C) - A \cdot 03 \]

\[ \theta = 2 \cdot (01 + 02 + 03) \]

\[ \theta = \frac{1}{1 + \omega} B = \frac{\beta}{A} B = \frac{\alpha}{A} C = \frac{\alpha}{A} D = \frac{\alpha}{A} \]

\[ E = \sqrt{\frac{2}{1 + \omega}} F = \sqrt{\frac{2A}{C}} \]

\[ \rho = E \cdot G = \gamma \sqrt{\frac{1}{1 + \omega}} \]

\[ u_{cn2n} = u_{cn3n} + 0.5 \]

\[ \alpha = C_p/C_s \]

\[ \beta = L_1/L_3 \]

\[ \theta_1, \theta_2, \theta_3 \]

\[ \theta = \omega T_s \]

\[ \omega = \omega', \sqrt{(1 + \alpha)/\alpha} \]

\[ \omega = \sqrt{L_1C_s} \]

With the solutions given above, (14), operating waveforms and converter gain can be obtained. Figure 3 shows typical waveforms for Mode 1 and Mode 2 in normalized form for a whole switching period.

The border of an interval is marked by a dotted vertical line. Figure 3 shows waveforms at the boundary between the two modes. The duration of the third interval (energy-transfer interval) has become zero and the converter behaviour is described by the energy-return/transfer interval and the commutation-interval. Figures 4 to 6 show operation diagrams - converter gain M versus normalized switching frequency f_s with normalized load resistance R as parameter for different values of \( \alpha \) and \( \beta \).

Variation of \( \alpha \) and \( \beta \) influences the converter behaviour significantly. Increasing \( \alpha \), i.e. enlarging the parallel-resonant capacitance \( C_p \), lets the series-parallel-loaded resonant-converter (SPRC) become a parallel-loaded one (PRC). Accordingly SPRC becomes a series-loaded resonant-converter (SRC) by decreasing \( \alpha \), i.e. enlarging the series-resonant capacitance \( C_s \).

\( \beta \) describes the distribution of the resonant inductance to the primary and tertiary transformer side. Concentrating the whole resonant inductance on the primary side, corresponding to \( \beta = 1.0 \), represents the ideal SPRC without leakage inductances analyzed by Kunze in the time and frequency domain. The operation diagram for \( \alpha = 1.0 \) and \( \beta = 0.999 \) is given in figure 4 (solid lines). The diamonds and X represent operating points of the ideal SPRC taken from the operation diagram of Kunze.

Decreasing \( \beta \) to zero, i.e. concentrating the resonant inductance on the tertiary side, leads to a PWM-converter between primary and secondary side additionally loaded with a resonant circuit on the tertiary side.

![Fig. 4 Operation diagram for α = 1.0, β = 0.999 (solid lines). X and ○ mark operating points from Kunze (1989)](image-url)
Fig. 5 Comparison of operation diagrams for $\alpha = 1.0$, $\beta = 0.9$ (solid lines) and $\alpha = 1.0$, $\beta = 0.999$ (X and ◦) ($\neq$ ideal SPRC)

Fig. 6 Operation diagrams for $\alpha = 1.0$, $\beta = 0.8$ (a) and $\alpha = 2.0$, $\beta = 0.9$ (b)

Fig. 7 Stress diagrams for the RMS-value of the inductor current in normalized form.

a) $\alpha = 0.5$, $\beta = 0.9$,  b) $\alpha = 1.0$, $\beta = 0.9$,
c) $\alpha = 1.0$, $\beta = 0.8$,  d) $\alpha = 2.0$, $\beta = 0.9$,
The deviation of the operation diagrams considering the leakage inductances from the ideal case is shown in figure 5 for \( \alpha = 1.0 \) and \( \beta = 0.9 \) (solid lines). Diamonds and X again mark operating points of the ideal converter.

- In the non-ideal case the maximum converter gain appears at higher switching frequencies caused by shifted converter input impedances.

- For \( Q \gtrsim 2.0 \) and higher switching frequencies the converter gain of the SPRC considering the leakage inductances is higher than for the ideal one. In the ideal case the capacitor voltage \( u_{CP} \) is the input voltage of the bridge rectifier. After a zero-crossing or at the end of the discontinuous-capacitor-voltage interval (see Kunze, 1989), \( u_{CP} \) increases in a sinusoidal way. In the non-ideal case (\( \beta < 1.0 \)), \( u_{12} \) is the input voltage of the rectifier diodes, jumping at the end of the commutation interval to an amplitude, determined by \( u_{CP} \) and the derivation of \( i_3 \) at this moment. Therefore a higher time-voltage product and consequently a higher converter gain results.

- For \( Q \gtrsim 2.0 \) and normalised switching frequencies closer to 1.0 the converter gain of the SPRCT is lower as compared with the ideal SPRC because the influence of the resonance step-up of the capacitor voltage \( u_{CP} \) at \( f_s \approx f_o \) is less significant.

- For \( Q \leq 1.0 \) the minimum of the converter input impedance is decreased for the SPRC considering the leakage inductances leading to a higher maximum converter gain.

The converter gain curves for \( Q \leq 0.5 \) are nearly the same as for \( Q = 0.5 \). Therefore no-load operation is possible.

Component stress

To optimize the converter design knowledge about the component stress is essential. In addition to the maximum ratings the following quantities are useful:

- RMS value of the inductor current, \( I_{L\text{RMS}} \), for the design and optimization of the resonant inductor,

- RMS value of the transistor current, \( I_{S\text{RMS}} \), to estimate the conduction losses,

- AV value of the transistor current, \( I_{S\text{AV}} \), to estimate the conduction losses, if bipolar transistors are applied,

- RMS value of the capacitor voltages, \( U_{C\text{RMS}}, U_{C\text{RMS}} \), for selection of appropriate capacitors,

- time-voltage product at the transformer terminals for its design.

Equations for the desired AV- and RMS-values and the time-voltage product can be derived from the general solutions leading to stress diagrams (normalized RMS- or AV-value versus normalized load resistance with converter gain as parameter). Figure 7 shows the normalized RMS-value of the resonant-inductor current, \( I_{L\text{RMS}} \), for different values of \( \alpha \) and \( \beta \) and typical load and converter gain ranges.

Increasing \( \alpha \) leads to stress diagrams insensitive to load variations in a wide range, see figure 7d. A poor efficiency characteristic for lighter loads results as for a parallel-loaded resonant-converter. A variation of \( \beta \) influences the magnitude of \( I_{L\text{RMS}} \), as can be seen in fig. 7b and 7c.

Beyond the component stress diagrams the operation diagrams have to be taken into consideration to optimize the converter design. A compromise between minimum component stresses – leading to small values of \( \alpha \) – and a small frequency range together with wide load and input voltage range – leading to larger values of \( \alpha \) – must be found. In any case \( \beta \) should be close to 1.

To verify the theoretical results realization of a SPRC with tertiary side resonance is in preparation considering the above mentioned optimization hints.

**CONCLUSIONS**

The state variables waveform, the converter gain and component stress diagrams of the series-parallel-loaded resonant-converter with tertiary side resonance operating above resonant frequency, derived from an analysis in the time domain reveal that the converter, utilizing a part of the transformer leakage inductances as resonant element, can be designed for wide load and input voltage range with a relative small frequency range and acceptable component stress by proper choice of the design parameters \( \alpha \) and \( \beta \).

It has been found that \( \alpha \approx 1.0 \) is a good compromise making use of the main advantages of the series-parallel resonant-converter (SRC) (e.g. good efficiency characteristic for lighter loads) and the parallel-loaded one (PRC) (e.g. reduced frequency range with wide load and input voltage range).

The inductance ratio \( \beta \) should be close to 1, corresponding to a small tertiary leakage inductance. Together with a small secondary leakage inductance a strong coupling between secondary and tertiary transformer winding results, leading to shorter commutation-interval durations and therefore to an increase of the time-voltage product for the output filter. A strong coupling between the two transformer windings does not make too much problems because isolation specifications need not to be considered. Thus, strong coupling leads to the projected reduction in blocking voltage of the rectifying power diodes on the secondary side which affects reduced conduction losses of the dominant loss generating components.

**REFERENCES**

