Ready for Fields&Waves?

Prof. Dr. Jens Förstner

September 27, 2016

The following questions cover a range of topics you should master (i.e. >75% correct answers) before starting the course Fields&Waves in the Electrical Systems Engineering programm at Paderborn University:

1. Evaluate
   (a) $\sin\frac{\pi}{2}$ =1
   (b) $\cos\frac{\pi}{2}$ =0
   (c) $\sin^2 x + \cos^2 x$ =1
   (d) $\exp(0)$ =1
   (e) $\exp(-\frac{\pi}{2}j)$ =-j
2. Express e^{jx} in terms of sin and cos (Euler's identity): $e^{jx} = \cos x + j\sin x$
3. Give the general real-valued solution of the ODEs
   (a) $\frac{d^2}{dt^2}y(t) = -\omega^2 y(t)$ (with $\omega \neq 0$): $y(t) = a\cos(\omega t) + b\sin(\omega t)$ or $a\sin(\omega t + \phi)$
   (b) $\frac{d}{dt}y(t) = -\gamma y(t)$ (with $\gamma \neq 0$): $y(t) = a\,e^{-\gamma t}$
4. Give the solution of the Fourier integral $g(\omega) = \int_{-\infty}^{\infty} g(t)\,e^{-j\omega t}dt$ for
   (a) $g(t) = \frac{d}{dt}f(t)$ (assume $f(\omega)$ is known): $\Rightarrow g(\omega) = -j\omega f(\omega)$
   (b) $g(t) = f(t)\,e^{j\omega_0 t}$ (assume $f(\omega)$ is known)$\Rightarrow g(\omega) = f(\omega - \omega_0)$
   (c) $g(t) = \sin(\omega_0 t) \ \Rightarrow g(\omega) = \pi\delta(\omega - \omega_0) - \pi\delta(\omega + \omega_0)$
5. Vector products, Give
   (a) the projection of a vector $\vec{a}$ on a normalized vector $\vec{n}$: $= \vec{a}\cdot\vec{n} = a\ \cos\phi$
   (b) the inner product $\vec{a}\cdot\vec{b}$ in cartesian coordinates: $= a_x b_x + a_y b_y + a_z b_z$
   (c) the length of a vector $\vec{a}$ using the innter product: $= \sqrt{\vec{a}\cdot\vec{a}}$
   (d) the vector product $\vec{a}\times\vec{b}$ in cartesian coordinates: $= \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix}$
6. Evaluate the following expressions (or mark if invalid):
   (a) grad 5 =0
   (b) curl 4 =Invalid expression
   (c) $\mathrm{grad}(x^2 + y^3) = \begin{pmatrix} 2x \\ 3y^2 \\ 0 \end{pmatrix}$
   (d) $\mathrm{curl}\,\mathrm{grad}\,\vec{v}(\vec{r})$ =0

(e) div $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$=3

(f) curl $\begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

7. State Stokes' and Gauss' theorems:

   (a) $\int_V \text{div}\,\vec{v}(\vec{r})\,dV = \oint_{\partial V} \vec{v}(\vec{r}) \cdot d\vec{a}$

   (b) $\int_A \text{curl}\,\vec{v}(\vec{r}) \cdot d\vec{a} = \oint_{\partial A} \vec{v}(\vec{r}) \cdot d\vec{s}$

8. Give the electrostatic potential of a point charge q located at the the position $\vec{s}$: $\varphi(\vec{r}) = \frac{q}{4\pi\epsilon_0|\vec{r}-\vec{s}|}$

9. Write down the four Maxwell equations (for material/medium, in differential form, SI units):

   (a) $\text{curl}\vec{E} = -\frac{d}{dt}\vec{B}$

   (b) $\text{curl}\vec{H} = \frac{d}{dt}\vec{D} + \vec{J}$

   (c) $\text{div}\vec{D} = \rho$

   (d) $\text{div}\vec{B} = 0$

10. Which electric and magnetic field components are continuous at an interface?B normal, E tangential

11. For a perfect electric conductor, the electric field strength

    (a) inside is:0

    (b) at the surface is:orthogontal on surface

12. In a medium give (in terms of the real-valued e.m. fields) the definitions of

    (a) the Poynting vector: $\vec{S} = \vec{E} \times \vec{H}$

    (b) the electromagnetic energy (in a volume V): $W = \frac{1}{2}\int_V (\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})\,dV$

13. Give the units (in SI) of

    (a) the electric field strength: $[\vec{E}] = V/m$

    (b) the magnetic flux denisity: $[\vec{B}] = Vs/m^2 = T$

    (c) the current density: $[\vec{J}] = A/m^2$

    (d) the charge density: $[\rho] = C/m^3 = As/m^3$