Optical and Unified Noise Figure, and Homodyne Noise Figure

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https://ieeexplore.ieee.org/document/10433655 (open access)

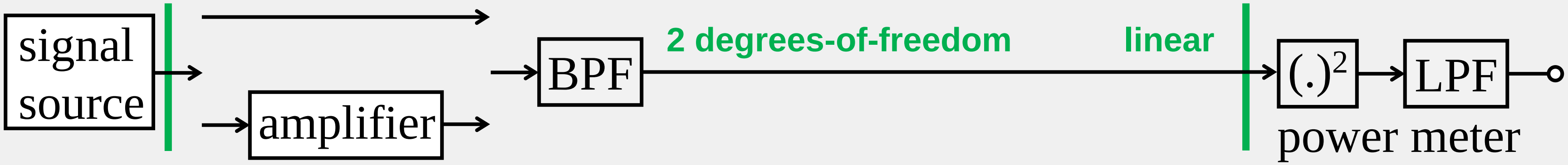
In earlier version https://ieeexplore.ieee.org/document/9915356 (open access)
kT needs to be replaced by $k'T$, see following viewgraph p. 29.

https://www.vde.com/resource/blob/2264668/06253acdbf74d710014e3ab507ac154f/do-propagating-lightwaves-contain-photons--data.pdf (open access) support material

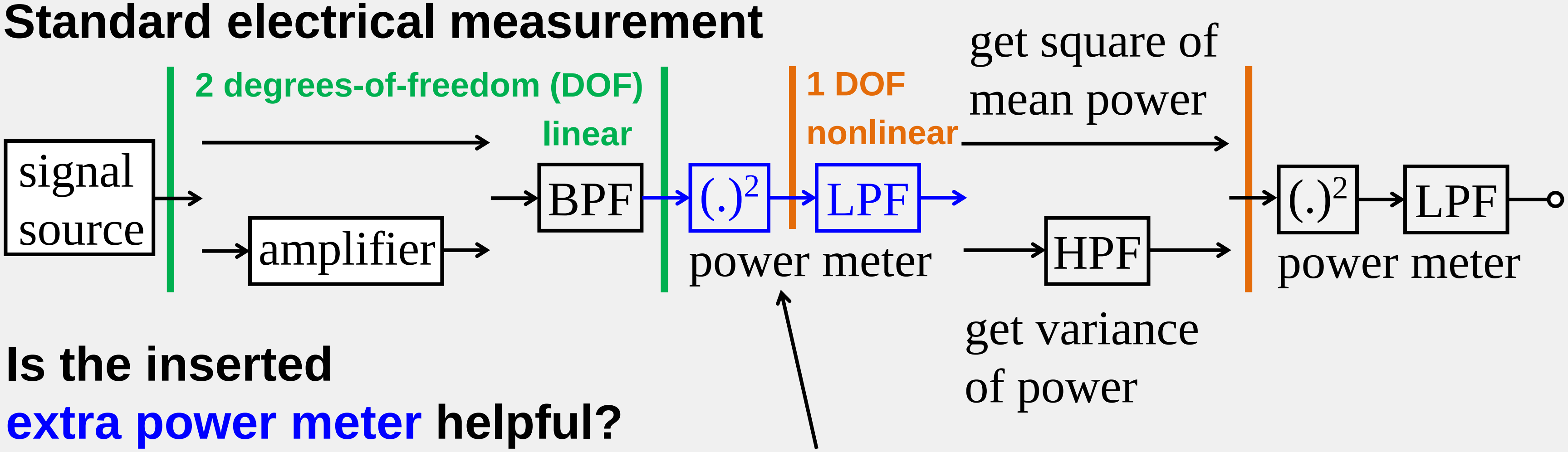
Overview

- <u>Motivation</u>
- <u>Noise figures in linear/coherent optical receivers</u>
  - <u>In-phase and quadrature noise figure = the noise figure</u>
  - In-phase (homodyne) noise figure = special case
- <u>Comparison of noise figures</u>
- Consistent unified noise figure
- Summary
- Appendix
  - Review of electrical noise figure
  - Review of prior optical and unified noise figures
  - Elimination of avoidable I&Q receiver noise
  - Review of photoelectron statistics

How to determine noise and gain properties of amplifier



Standard electrical measurement



Is the inserted
extra power meter helpful?

Probably not. Now this is a photodiode
and we are talking about optical signals!

$$G = e^{(a-b)t} = e^{(a-b)z/v_g}$$ gain

$$\mu = n_{sp}(G-1)$$ mean number of detectable output noise photons per mode

$$n_{sp} = \frac{a}{a-b}$$ spontaneous emission factor

$$P_{n,out}\tau = \mu hf = G\tilde{\mu}hf$$ mean output noise energy per mode

(traditional optical NF)

Power, gain, loss must be redefined if F_{pnf} is valid NF!

$$F=\frac{SNR_{in}}{SNR_{out}}=\frac{P_{s,in}P_{n,out}}{P_{n,in}P_{s,out}}=\frac{\text{noise gain}}{\text{signal gain}}$$

Science is **systematic** and **exact** and **does not tolerate contradictions**! Unit definitions **must not** depend on measurement method or f! F_{pnf} **implies**:

$$\frac{G^2\langle n\rangle^2}{\langle n\rangle^2}=G^2$$ **is** the signal „gain"! ⇔ Noise gain depends on signal; „powers" cannot be added; linearity condition is violated. ϟ ⇔ No more (optical) dB are allowed; „gain" must be given in „electrical" dB! ϟ ⇔ 1550 nm fiber loss is no longer 0.2 dB/km; „loss" must be stated as 0.4 dB/km! ϟ

Thermal power meter can replace photodiode and allows going to low f.

⇔ Any (electrical, optical) amplifier with 20 dB gain has 40 dB „gain"! ϟ

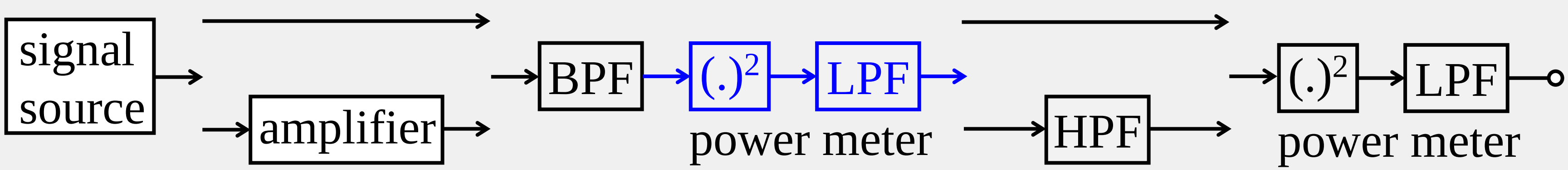
⇔ All „powers" (optical, electrical, thermal, mechanical) must be ~ squared powers, since all powers can be converted into thermal power and compared! ϟ ⇔ work = sqrt(„power")·time ϟ

squared mean of power or photoelectrons

$$SNR_{pnf,in}=\frac{\langle n\rangle^2}{\langle n\rangle}$$

variance of photoelectrons

$$SNR_{pnf,out}=\frac{G^2\langle n\rangle^2}{G\langle n\rangle+n_{sp}\left(2(G-1)G\langle n\rangle+\ldots\right)}$$



Problem introduction

- **Electrical** noise figure (NF) is standardized since many decades.
- Traditional **optical** noise figure F_{pnf} was defined in 1990ies, for optical direct detection receivers (DD RX). Problematic aspects, in **conflict** with electrical NF:
  - Optical signals have in-phase and quadrature components, like electrical signals and RX. But an optical DD RX **suppresses** phase information. ϟ

  ⇒ NF = 2 for ideal optical amplifier, whereas NF = 1 for ideal electrical amplifier.
  - „Power" in signal-to-noise (SNR) ratio calculation is ~ square of photocurrent in optical DD RX. Photocurrent is ~ optical power ~ **square** of field amplitude. SNR „power" is ~ 4th power of field amplitude ~ **square** of power. ϟ

    Incompatible with ~150 years of science: $P = U^{2}/R$, not $P \sim U^{4}$.
  - **Noise happens on a field basis.** Power measurement conceals fields!
  - Ideal DD RX for intensity modulation with / without ideal optical amplifier needs 38 / 10 photoelectrons/bit for bit error ratio = 10^{-9}. Ideal DD RX for differential phase shift keying: 20 / 20 photoelectrons/bit. $38/10 \neq F_{pnf} = 2 \neq 20/20$ **!**
  - Optical: **Non**linear DD RX; **non**-Gaussian noise; amplifier NF depends on power and bandwidths. Electrical: Linear RX; Gaussian noise; constant NF.
  - Unification of all prior optical NF with electrical NF is contradictory (pp. 44-49).

Noise figures until year 2000

Stated lower NF limits apply only for ideal amplifiers with high gain and 2 available quadratures! Expectation value of equivalent input-referred detectable noise photons per mode is $\tilde{\mu} = n_{sp}(1-1/G)$. Marked: Aspects that are problematic in my opinion

F_e = the NF [1], more precisely the electrical version of the NF, defined or understood as the **SNR degradation factor** ≥1 in a **linear** system with **2 available receiver (RX) quadratures**, i.e. 1 mode, standardized in 1960ies by H. Haus

$F_{pnf} = 1/G + 2\tilde{\mu}$ = traditional optical NF ≥2 of detected photon number fluctuation, defined by E. Desurvire for intensity modulation with direct detection (IM/DD), which is a nonlinear system that keeps only 1 quadrature (or degree-of-freedom)

$F_{ase} = 1 + \tilde{\mu}$ = optical NF ≥2 and unified NF, defined and intended by H. Haus for amplified spontaneous emission in a linear system with 2 available RX quadratures, but not equal to its optical SNR degradation factor

$F_{fas} = 1/G + 2\tilde{\mu}$ = optical NF ≥2 with conflicting definitions by H. Haus, in the latest version also a unified NF, slightly wrong regarding thermal noise at high f, linear with seemingly 1 available RX quadrature in optical domain, meant to become F_e in electrical domain where there are 2 available RX quadratures.

F_{fas} implies variable number of RX quadratures. For 1 available RX quadrature, special NF names would be needed, showing that F_{pnf}, F_{fas} are not optical equivalents of F_e.

Noise figures presented here

$F_{o,IQ} = 1/G + \tilde{\mu}$ = the optical NF ≥1 derived, in full agreement with F_e definition, as SNR degradation factor in a linear system with 2 available RX quadratures

F_{IQ} = the NF ≥1 for all f, with the limit cases of optical $F_{o,IQ}$ and electrical F_e . Important for very cool mm wave / THz systems and possibly for very hot far infrared systems.

$F_{o,I} = 1/G + 2\tilde{\mu}$ = optical homodyne NF ≥2, defined for the case of 1 available RX quadrature, SNR degradation factor in such linear system, equal to final version of optical F_{fas}, equal in value to F_{pnf}.

F_I = homodyne NF for all f, with the limit cases of optical homodyne $F_{o,I}$ and electrical homodyne, which is the same as F_e. Probably not important, since electrical homodyne RX and amplifiers don't have intrinsic noise advantages over their I&Q counterparts.

A NF with an aspect that differs from F_e needs a special denomination (homodyne, optical).

I think a NF which strongly differs (nonlinear, no SNR degradation factor) from F_e should not be called a NF.

Fields in coherent optical I&Q receiver

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\mathbf{e}_1 e^{j\omega t}$$

$$P =: |\mathbf{E}|^2 \qquad I = RP = e/(hf)\cdot P$$

Power (for simplicity) Photocurrent

v_1, v_2 zero-mean independent Gaussian

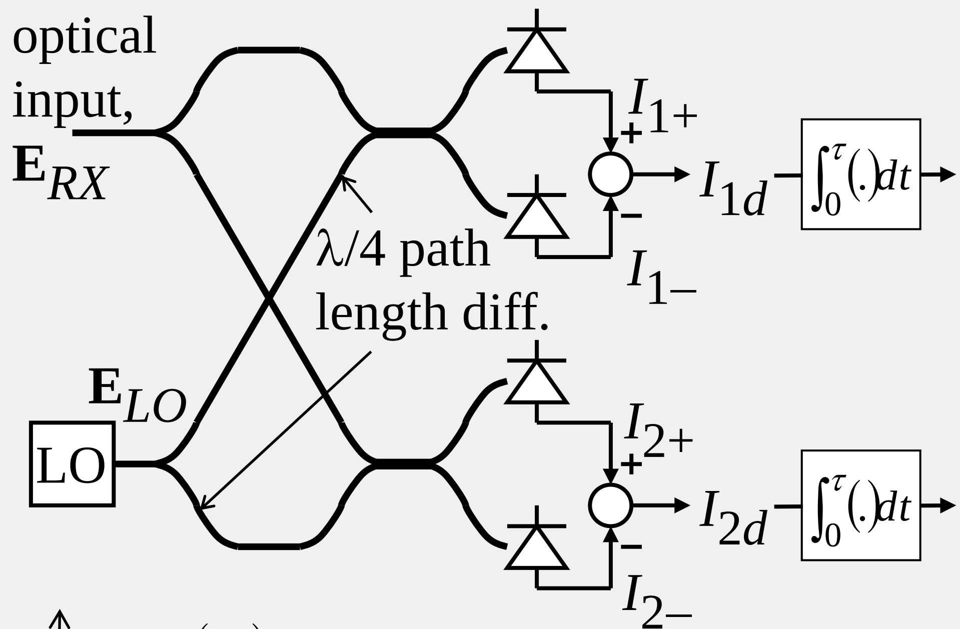
$$\langle v_1^2 \rangle = \langle v_2^2 \rangle = \sigma^2 = 1$$

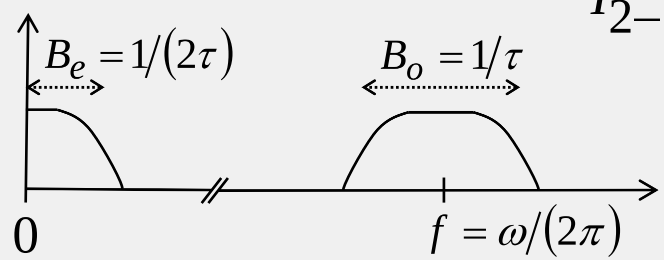
(see pp. 52-61)

$\mathbf{e}_1$ normalized field (polarization) vector

Optical signal is linearly downconverted to baseband. Local oscillator (LO) is a strong unmodulated laser with (essentially) the same frequency as the received signal.

2 available quadratures = 1 available mode





baseband spectrum

optical spectrum

Baseband I&Q RX is not mandatory!
Heterodyne RX with image rejection filter gives the same results!

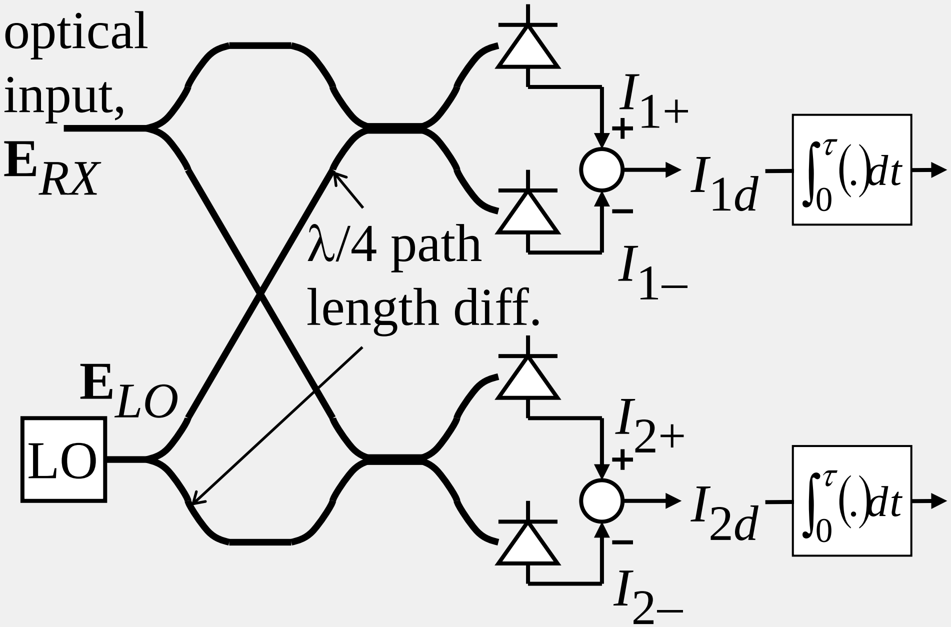
Photocurrents in coherent optical I&Q receiver ...

$$\mathbf{E}_{RX} = \sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$P =: |\mathbf{E}|^2 \qquad I = RP = e/(hf)\cdot P$$

(In practice, optical frequencies of signal and unmodulated local oscillator may differ a bit, causing the complex plane of I_{1d} and I_{2d} to rotate at the difference frequency.)



4 detected photocurrents

$$I_{1\pm} = R\left|\pm\mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + P_{LO}\right)$$

$$I_{2\pm} = R\left|\pm\mathbf{E}_{RX}/2 + j\,\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + P_{LO}\right)$$

...and their differences and sums

$$I_{1d} = I_{1+} - I_{1-} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

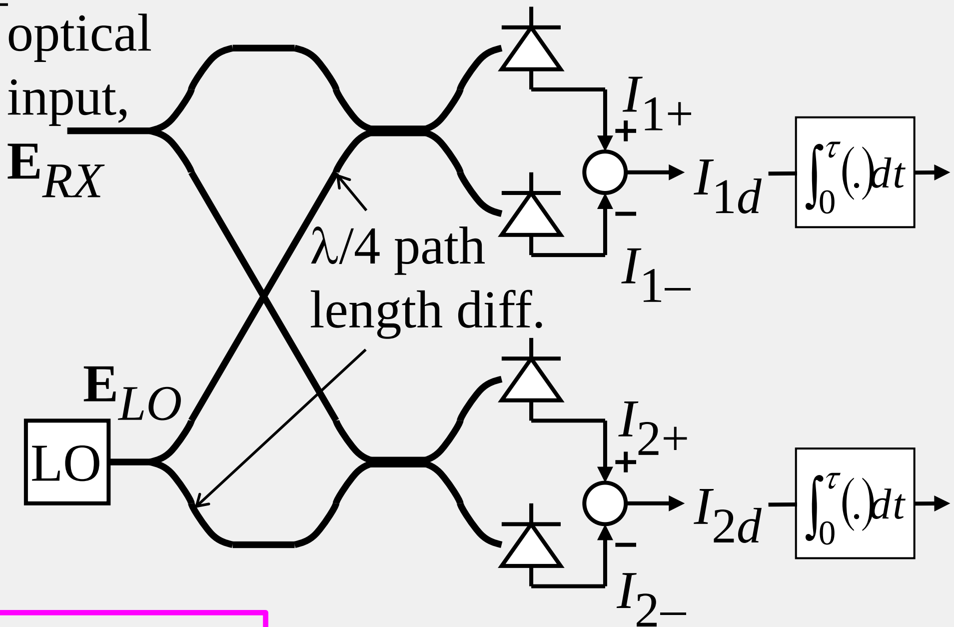
$$I_{2d} = I_{2+} - I_{2-} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = I_{1+} + I_{1-} = RP_{LO}/2$$

$$I_{2s} = I_{2+} + I_{2-} = RP_{LO}/2$$

Differences and sums of photocurrents

Neglect for $P_{LO} \rightarrow \infty$



Cancel in subtraction

4 detected photocurrents

$$I_{1\pm} = R\left|\pm\mathbf{E}_{RX}/2 + \mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}} + P_{LO}\right)$$

$$I_{2\pm} = R\left|\pm\mathbf{E}_{RX}/2 + j\mathbf{E}_{LO}/2\right|^2$$

$$= \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2v_2\sqrt{P_n/2}\sqrt{GP_{LO}} + P_{LO}\right)$$

SNR in coherent optical I&Q receiver

$$I_{1d} = R\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right)\sqrt{GP_{LO}}$$

$$I_{2d} = Rv_2\sqrt{P_n/2}\sqrt{GP_{LO}}$$

$$I_{1s} = RP_{LO}/2$$

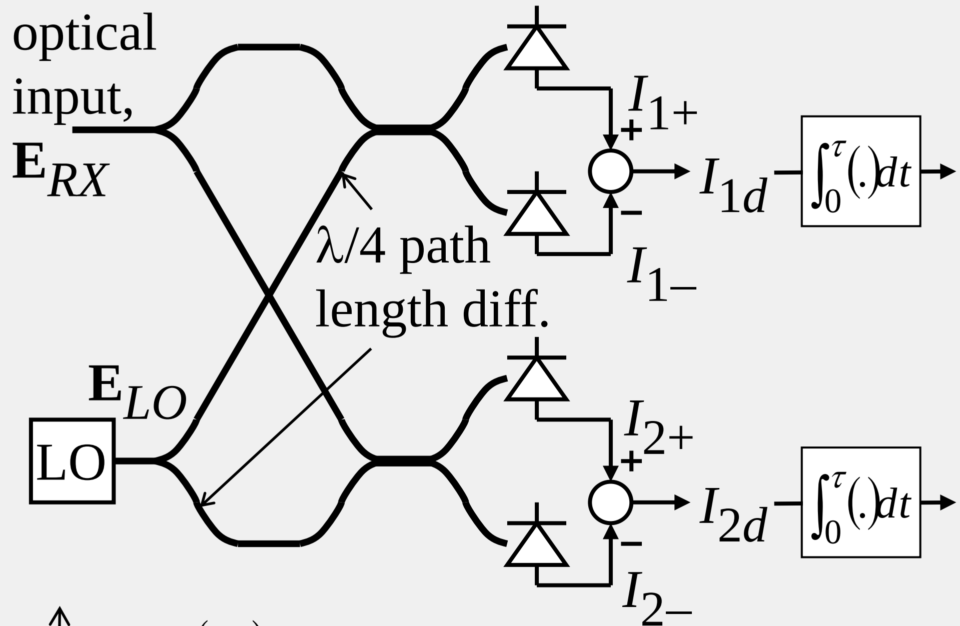
$$I_{2s} = RP_{LO}/2$$

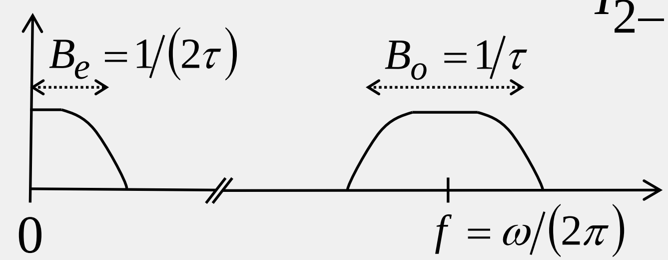
Pure Gaussian PDFs of interference + field noises!

Shot noise PSD: $2eI_{1s}$, $2eI_{2s}$

Optical bandwidth: $B_o = 2B_e = 1/\tau$

Equivalent amplifier input noise PSD per mode: $\tilde{\mu}hf = P_n/B_o$





For SNR calculation take either noise in 1 mode or (like I do it) in 1 quadrature! (Factor 2 cancels in NF calculation.)

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO} G\tilde{\mu}hfB_o/2 + eRP_{LO}B_e} = \frac{P_S\tau}{(\tilde{\mu} + 1/G)hf/2}$$

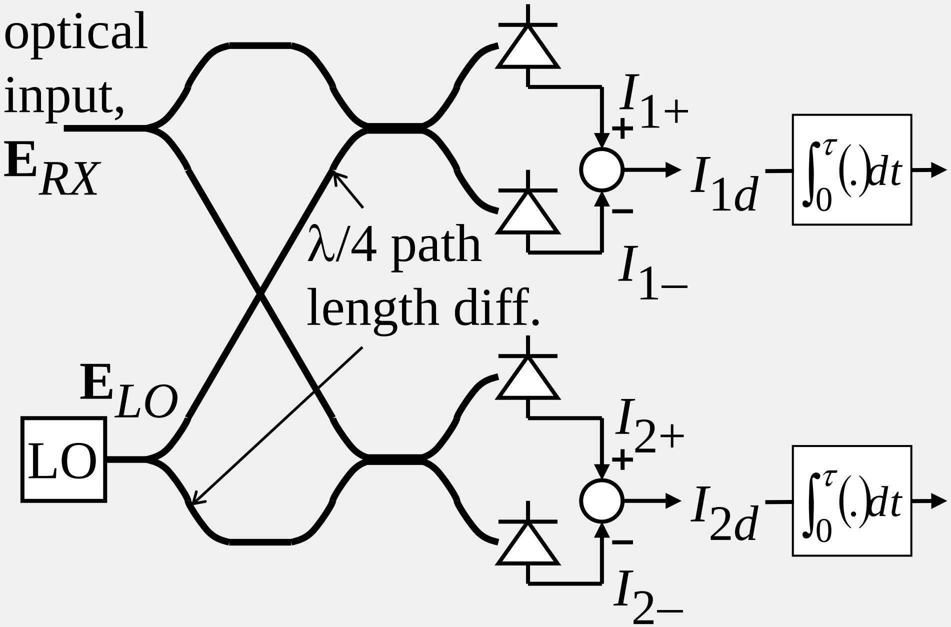
Optical I&Q noise figure (or heterodyne with image rej.)

$$SNR_{o,IQ,out} = \frac{P_S \tau}{(\tilde{\mu} + 1/G) hf/2}$$

No amplifier, $G = 1, \quad \tilde{\mu} = 0$:

$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2}$$

$$\frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} =$$



$$F_{o,IQ} = \tilde{\mu} + 1/G = n_{sp}(1 - 1/G) + 1/G$$

$$= 1 + (n_{sp} - 1)(1 - 1/G) \geq 1$$

$F_{o,IQ}$ obeys the usual electrical NF definition, is SNR degradation factor; powers ~ squares of amplitudes; 2 available RX quadratures; linearity; ideal NF = 1; pure Gaussian noise!

$$F_{o,IQ} = (F_{pnf} - 1/G)/2 + 1/G$$

$$F_{pnf} = 2(F_{o,IQ} - 1/G) + 1/G$$

$$F_{o,IQ} \approx F_{pnf}/2$$

Conversion formulas

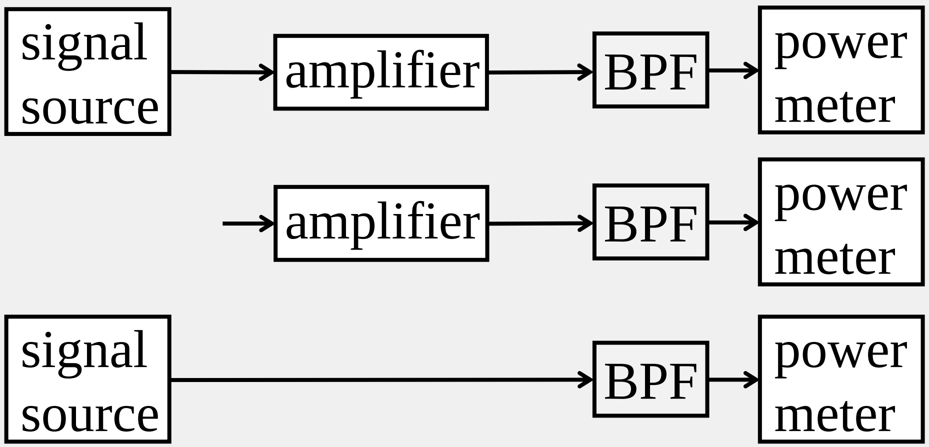
≈ 3 dB difference for large G

Measure optical I&Q noise figure with power meter

Optical amplifier must be loaded with extra optical signal power at other times/frequencies/polarization in order to keep G, $\tilde{\mu}$ constant.

Usually there are $p=2$ polarization modes. $p=1$ requires inserted polarizer.

Of course, offset (dark current) must be subtracted from power meter readout.



$$P_1 = pG\tilde{\mu}hfB_o + GP_S$$

$$P_2 = pG\tilde{\mu}hfB_o$$

$$P_S$$

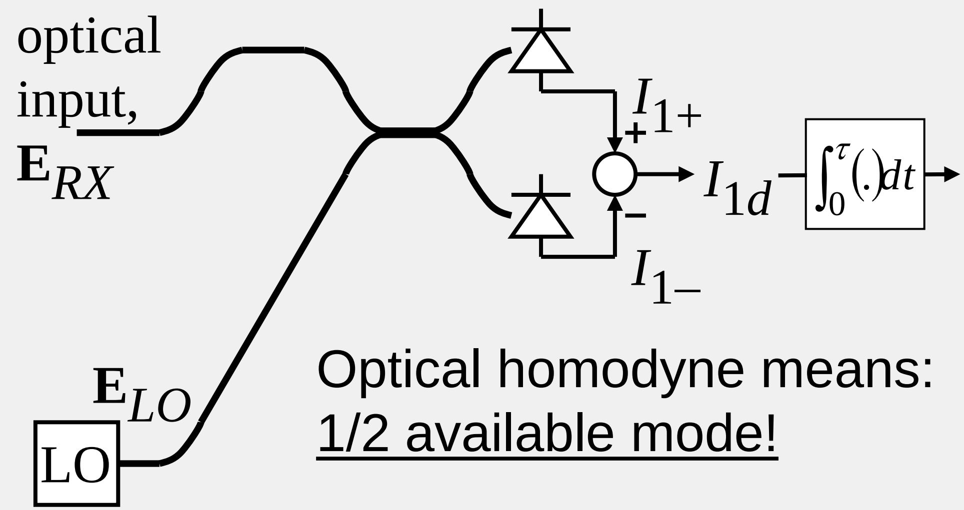
$$G = \frac{P_1 - P_2}{P_S} \qquad \tilde{\mu} = \frac{P_2}{pGhfB_o}$$

$$F_{o,IQ} = \tilde{\mu} + 1/G$$

$F_{o,IQ}$ and all other optical NF can be determined from simple optical power measurements.

Optical I noise figure (true homodyne; special case)

In such cases, phase locking is required between signal and LO or detector!
No power splitting ⇒ In equations multiply each of $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ by 2.



$$F_{o,I} = 2\tilde{\mu} + 1/G \qquad (= F_{fas} = F_{pnf})$$

$$= 1 + (2n_{sp} - 1)(1 - 1/G) \geq 1$$

Optical homodyne means: 1/2 available mode!

$F_{o,I}$ is similar to $F_{o,IQ}$ and F_e, but only 1 quadrature is available.
Lowest $F_{o,I} \to 1$ for $G \to 1$. Ideal $F_{o,I} = 2$ at $G \to \infty$. Why?

Optical amplifier is not special! RX is special: 1 quadrature & detection (= shot) noise!

Without optical amplifier, true homodyne RX is twice as sensitive as I&Q RX because P_{RX} is not split. But with optical amplifier having $G \to \infty$, output power splitting like in the I&Q RX cannot have an SNR effect. So, behind the amplifier the homodyne RX "must" have the worse sensitivity of the I&Q RX. Amplifier halves homodyne SNR!

Phase-sensitive degenerate parametric optical amplifier passes only 1 quadrature and has ideal $F_{o,I} = 1$ and $F_{o,IQ} = 1/2$ (converts I&Q into more sensitive homodyne).

Zero-point fluctuations can explain/replace shot noise

Shot noise can be derived either way (but only 1 way at a time, not 2 ways at a time):

- Semiclassical theory: Poisson distribution of photoelectrons has one-sided photocurrent power spectral density (PSD) $\boxed{2eI}$.
- Zero-point fluctuations interfere with signal and cause shot noise PSD $\boxed{2eI}$.

Let us define field such that power is $P := |\mathbf{E}|^2$. Observation time is $\tau = 1/B_o$.

Zero-point fluctuations have mean energy $W = P\tau$ equal to $hf/2$ per mode:

$$\mathbf{E}_0 = (u_1 + ju_2)\underline{\mathbf{e}}_1 e^{j\omega t} \qquad \sigma_{u1}^2 = \sigma_{u2}^2 = hf/(4\tau) \qquad |\underline{\mathbf{e}}_1| = 1$$

Signal field: $\mathbf{E}_S = \sqrt{P_S}\,\mathbf{e}_1 e^{j\omega t}$ Total field: $\mathbf{E}_S + \mathbf{E}_0$

Expected number of photoelectrons: $n_{S+0} = |\mathbf{E}_S + \mathbf{E}_0|^2 \tau/(hf)$

$$= \left(|\mathbf{E}_S|^2 + 2\,\mathrm{Re}\left(\mathbf{E}_0^+ \mathbf{E}_S\right) + |\mathbf{E}_0|^2\right)\tau/(hf) \approx \left(P_S + 2u_1\sqrt{P_S}\right)\tau/(hf)$$

Mean: $\langle n_{S+0}\rangle = \dfrac{P_S\tau}{hf}$ Variance: $\sigma_{nS+0}^2 = \dfrac{hf}{4\tau} P_S \dfrac{2^2\tau^2}{h^2 f^2} = \dfrac{P_S\tau}{hf} = \langle n_{S+0}\rangle$

$$I = RP = \boxed{\frac{e}{hf}}P \qquad \langle I_{S+0}\rangle = \langle n_{S+0}\rangle \frac{e}{\tau} \qquad \sigma_{IS+0}^2 = \sigma_{nS+0}^2 \frac{e^2}{\tau^2} = \boxed{2e\cdot\boxed{\frac{e}{hf}}P_S}\cdot\boxed{\frac{1}{2\tau}}$$

one-sided electrical bandwidth

I&Q NF derived with zero-point fluctuations (1)

$$\mathbf{E}_{RX1} = \left[\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_{11} + ju_{12})\right]\mathbf{e}_1 e^{j\omega t}$$

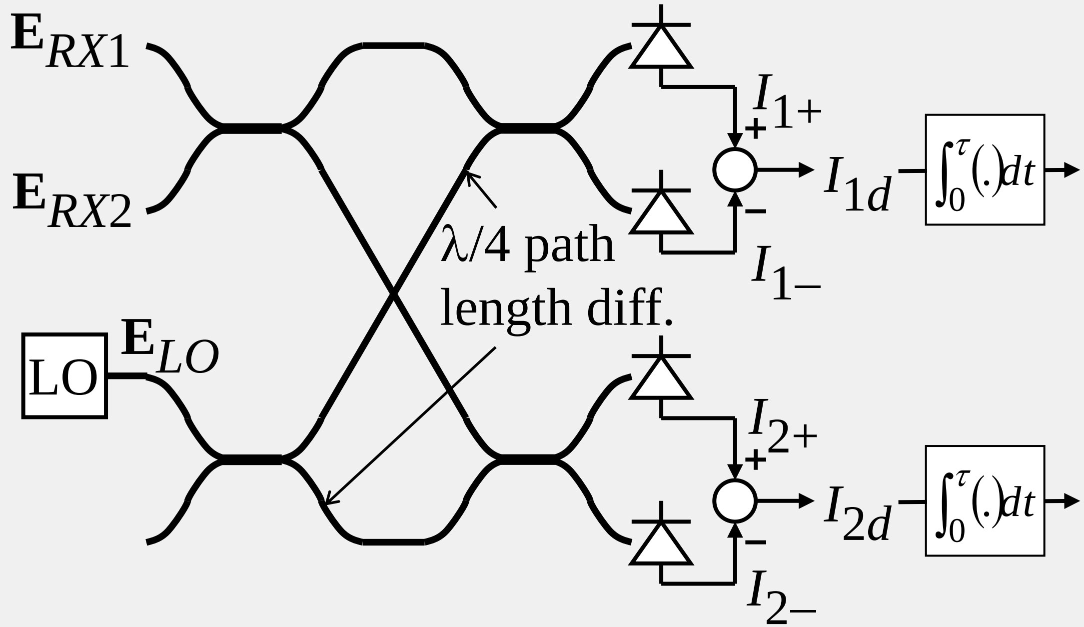
$$\mathbf{E}_{RX2} = (u_{21} + ju_{22})\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$

$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2$$

$$w_1 = u_{11} + u_{21} \qquad w_2 = u_{12} - u_{22}$$

$$2\left\langle u_{ij}^2 \right\rangle = \left\langle w_k^2 \right\rangle = \frac{hf}{2\tau} \qquad \left\langle v_k^2 \right\rangle = 1$$



Zero-point fluctuations occur at both signal ports. Mean power of zero-point fluctuations is neglected for simplicity.

$$I_{1\pm} = R\left|\pm(\mathbf{E}_{RX1} + \mathbf{E}_{RX2})/2 + \mathbf{E}_{LO}/2\right|^2$$

$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

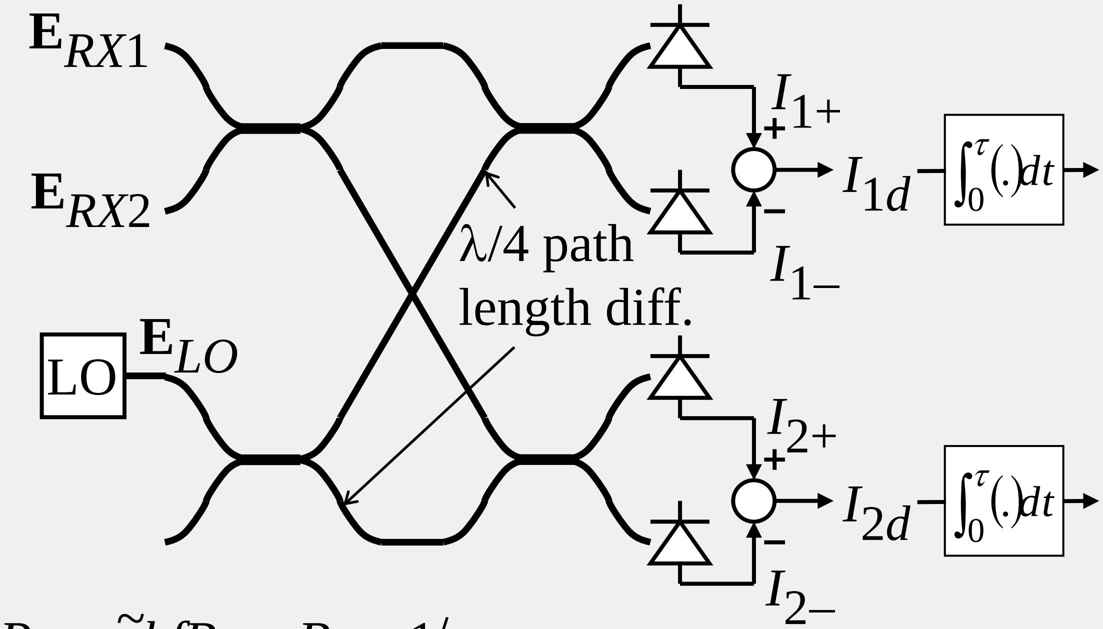
$$I_{2\pm} = R\left|\pm(\mathbf{E}_{RX1} - \mathbf{E}_{RX2})/2 + j\mathbf{E}_{LO}/2\right|^2$$

$$\approx \frac{R}{4}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}} + P_{LO}\right)$$

I&Q NF derived with zero-point fluctuations (2)

The 2 LO ports also carry zero-point fluctuations. But these cancel upon subtraction of photocurrents.

$$I_{1d} = I_{1+} - I_{1-}$$
$$= R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + w_1\right)\sqrt{P_{LO}}$$
$$I_{2d} = I_{2+} - I_{2-}$$
$$= R\sqrt{G}\left(v_2\sqrt{P_n/2} + w_2\right)\sqrt{P_{LO}}$$



$$\left\langle w_k^2 \right\rangle = \frac{hf}{2\tau} \qquad \left\langle v_k^2 \right\rangle = 1 \qquad P_n = \tilde{\mu} hf B_o \quad B_o = 1/\tau$$

$$SNR_{o,IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_{I_{1d}}^2} = \frac{R^2 P_{LO} G P_S}{R^2 P_{LO}\left(G P_n/2 + hf/(2\tau)\right)} = \frac{G P_S}{G\tilde{\mu} hf B_o/2 + hf B_o/2} = \frac{P_S \tau}{\left(\tilde{\mu} + 1/G\right) hf/2}$$

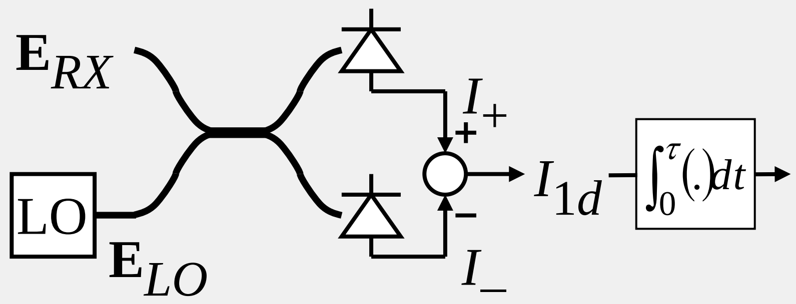
$$SNR_{o,IQ,in} = \frac{P_S \tau}{hf/2} \qquad \frac{SNR_{o,IQ,in}}{SNR_{o,IQ,out}} = F_{o,IQ} = \tilde{\mu} + 1/G$$

Same result as when derived with Poisson photoelectron distribution.

Homodyne NF derived with zero-point fluctuations

$$\mathbf{E}_{RX} = \left(\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_1 + ju_2)\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j\omega t}$$



$$I = RP = e/(hf)\cdot P \qquad P =: |\mathbf{E}|^2 \qquad \left\langle u_k^2 \right\rangle = \frac{hf}{4\tau} \qquad \left\langle v_k^2 \right\rangle = 1$$

$$I_\pm = R\left|\pm \mathbf{E}_{RX}/\sqrt{2} + \mathbf{E}_{LO}/\sqrt{2}\right|^2$$

Optical homodyne means: <u>1/2 available mode!</u>

$$\approx \frac{R}{2}\left(G\left(P_S + 2v_1\sqrt{P_S P_n/2} + \left(v_1^2 + v_2^2\right)P_n/2\right) \pm 2\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}} + P_{LO}\right)$$

$$I_d = I_+ - I_- = 2R\left(\sqrt{G}\left(\sqrt{P_S} + v_1\sqrt{P_n/2}\right) + u_1\right)\sqrt{P_{LO}}$$

$$SNR_{o,I,out} = \frac{\overline{I_d}^2}{\sigma_{I_d}^2} = \frac{4R^2 P_{LO} G P_S}{4R^2 P_{LO}\left(GP_n/2 + hf/(4\tau)\right)} = \frac{2GP_S}{2G\tilde{\mu} hfB_o/2 + hfB_o/2} = \frac{2P_S\tau}{\left(2\tilde{\mu} + 1/G\right)hf/2}$$

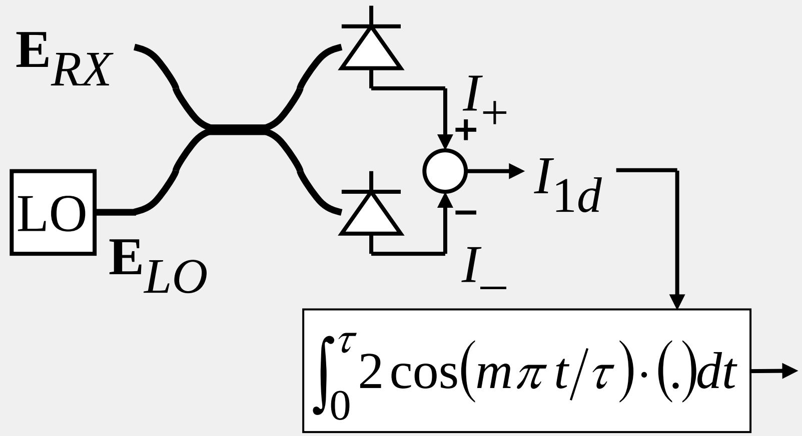
$$SNR_{o,I,in} = \frac{2P_S\tau}{hf/2} \qquad \frac{SNR_{o,I,in}}{SNR_{o,I,out}} = F_{o,I} = 2\tilde{\mu} + 1/G$$

Same result as when derived with Poisson photoelectron distribution.

Heterodyne NF

$$\mathbf{E}_{RX} = \left(\sqrt{G}\left(\sqrt{P_S} + (v_1 + jv_2)\sqrt{P_n/2}\right) + (u_1 + ju_2)\right)\mathbf{e}_1 e^{j\omega t}$$

$$\mathbf{E}_{LO} = \sqrt{P_{LO}}\,\mathbf{e}_1 e^{j(\omega - m\pi/\tau)t} \qquad (m = 1, 2, 3 \ldots)$$



Integration result of homodyne can be seen as sampled output signal of filter with rectangular impulse response 1 of duration τ.

Output signal is scaled by $\int_0^\tau 1 \cdot dt = \tau$.

Energy of impulse response: $\int_0^\tau 1^2 \cdot dt = \tau$

Heterodyne:

- I_{1d} appears multiplied by $\cos(m\pi t/\tau)$.
- Demodulate with carrier $2\cos(m\pi t/\tau)$.

Output signal is same as for homodyne, and same as without demodulator and with impulse response $2\cos(m\pi t/\tau)$ of duration τ. Energy in impulse response and noise are doubled: $\int_0^\tau (2\cos(m\pi t/\tau))^2 dt = 2\tau \quad \Rightarrow$

$$SNR_{o,het,out} = \frac{SNR_{o,I,out}}{2} = \frac{P_S \tau}{(2\tilde{\mu} + 1/G)hf/2}$$

$$F_{o,het} = 2\tilde{\mu} + 1/G = F_{o,I}$$

But with optical image rejection (ir) filter, amplifier noise ($2\tilde{\mu}$) is halved ($\tilde{\mu}$):

$$SNR_{o,het+ir,out} = SNR_{o,IQ,out}$$

$$F_{o,het+ir} = \tilde{\mu} + 1/G = F_{o,IQ}$$

Zero-point fluctuations or Poisson photoelectron distribution: same result.

Why does IM/DD with $P_S \to \infty$ behave like homodyne?

LO shot noise in a true homodyne RX and shot noise in an IM/DD RX with strong input signal can be taken into account by co-polarized in-phase zero-point fluctuations. Also after amplification it suffices to consider co-polarized in-phase noise. If one squares the output signal of the homodyne RX this becomes equivalent to an IM/DD RX.

$$P := |\mathbf{E}|^2 \qquad |\mathbf{E}| = \left|\sqrt{P_S} + u_1\right| \qquad P \approx P_S + 2u_1\sqrt{P_S}$$

True optical homodyne

$$SNR_{in} = \frac{P_{S,in}}{\left\langle u_{1,in}^2 \right\rangle}$$

$$SNR_{out} = \frac{P_{S,out}}{\left\langle u_{1,out}^2 \right\rangle}$$

$$F_{o,I} = \frac{P_{S,in}\left\langle u_{1,out}^2 \right\rangle}{P_{S,out}\left\langle u_{1,in}^2 \right\rangle}$$

Optical IM/DD

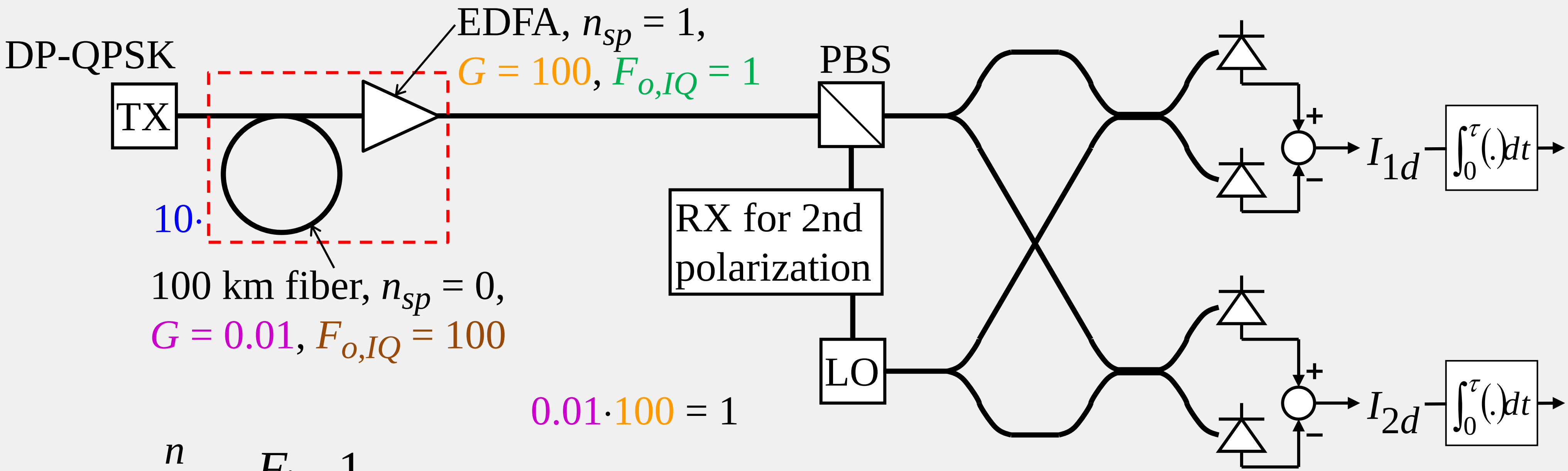
$$SNR_{in} = \frac{P_{S,in}^2}{4\left\langle u_{1,in}^2 \right\rangle P_{S,in}}$$

$$SNR_{out} = \frac{P_{S,out}^2}{4\left\langle u_{1,out}^2 \right\rangle P_{S,out}}$$

$$F_{pnf} = \frac{P_{S,in}^2 \, 4\left\langle u_{1,out}^2 \right\rangle P_{S,out}}{P_{S,out}^2 \, 4\left\langle u_{1,in}^2 \right\rangle P_{S,in}} = F_{o,I}$$

Essentially the same, namely $|\mathbf{E}| = \sqrt{P}$ and insertion of a square-root device at the output of an IM/DD receiver, has been explained by B.M. Oliver, “Thermal and Quantum Noise”, Proc. of the IEEE, 1965, pp. 436-454. At low f (thermal source noise) we get $F_{pnf} = F_e \geq 1$.

Example: NF of amplified trunk line, 1000 km, DP-QPSK



$$F-1=\sum_{i=1}^{n}\frac{F_i-1}{\prod_{k=1}^{i-1}G_k} = 10\cdot((100-1)+(1-1)/0.01) = 990$$

<u>All fibers and amplifiers may be combined to form one single amplifier.</u>
Dual-polarization QPSK transmission require for instance 18 detectable photons/bit. 1550 nm, 100 Gb/s, 25 Gbaud ⇒ –36.4 dBm is needed at input of ideal DP-QPSK I&Q RX. Due to $\boldsymbol{F_{o,IQ}}$ **= 991 ⇒ 30 dB** a DP TX power of –6.4 dBm is required. Practically, $n_{sp} > 1$, RX sensitivity may differ (FEC, wider bandwidth, intersymbol interference, thermal noise, …).

NF of distributed optical amplifier (e.g. Raman)

$$F-1=\sum_{i=1}^{n}\frac{F_i-1}{\prod_{k=1}^{i-1}G_k}=\int G^{-1}(\tau)\,d\left(F_\mathrm{i}(\tau)-1\right)$$

$$F_{o,IQ}=\tilde{\mu}+1/G=1+\left(n_{sp}-1\right)\left(1-1/G\right)$$

$$d\left(F_{o,IQ,\mathrm{i}}(\tau)-1\right)/d\tau=b(\tau)$$

$$F_{o,IQ}=1+\int_0^{\tau_g}b(\tau)e^{-\int_0^{\tau}(a(\vartheta)-b(\vartheta))d\vartheta}d\tau$$

$$F_{o,I}=2\tilde{\mu}+1/G=1+\left(2n_{sp}-1\right)\left(1-1/G\right)$$

$$d\left(F_{o,I,\mathrm{i}}(\tau)-1\right)/d\tau=a(\tau)+b(\tau)$$

$$F_{o,I}=1+\int_0^{\tau_g}\left(a(\tau)+b(\tau)\right)e^{-\int_0^{\tau}(a(\vartheta)-b(\vartheta))d\vartheta}d\tau=F_{pnf}$$

Whole fiber is Raman amplifier! Amplifier must be behind point where SNR_{in} is determined. ⇔ RX input must be before Raman fiber. ⇒ Raman amplifier has NF > 0 dB!

Group delay of amplifier: τ_g

Excess NF of infinitesimally short amplifier (i): $d\left(F_\mathrm{i}(\tau)-1\right)\to 0$

Spontaneous emission factor: $n_{sp}=\frac{a}{a-b}$

$$G(\tau)=e^{\int_0^{\tau}(a(\vartheta)-b(\vartheta))d\vartheta}$$

= total gain preceding infinitesimal amplifier (i) with gain

$$G_\mathrm{i}(\tau)=G(\tau)/G(\tau-d\tau)=1+\left(a(\tau)-b(\tau)\right)d\tau$$

Temporal can be replaced by longitudinal integrations:

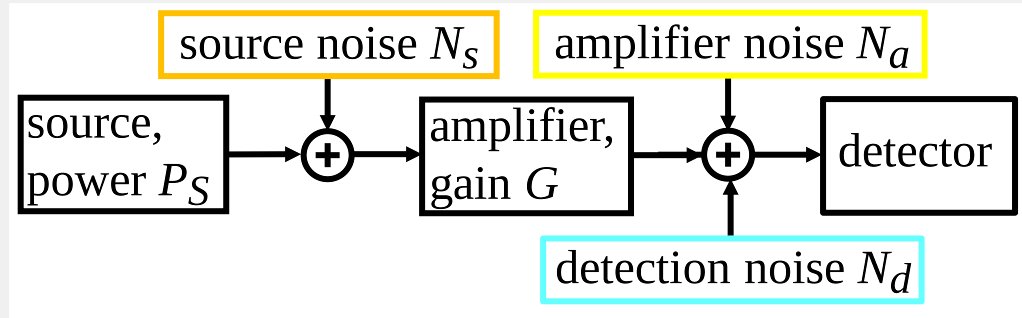
$$z=\int v_g\,d\tau \qquad dz=v_g\,d\tau$$

Structure of noise figure which fulfills Friis‘ formula

$$SNR_{in} = \frac{P_S}{N_s + N_d}$$

$N_s \sim kTB$?

$N_d \sim hfB$?



$$SNR_{out} = \frac{GP_S}{GN_s + N_d + N_a}$$

$$F = \frac{GN_s + N_d + N_a}{G(N_s + N_d)} = A + \frac{1-A}{G} + B$$

source noise fraction

$$A = \frac{N_s}{N_s + N_d}$$

added noise fraction

$$B = \frac{N_a}{G(N_s + N_d)}$$

Device cascade:

$$SNR_{out} = \frac{G_1 G_2 P_S}{G_1 G_2 N_s + N_d + G_2 N_{a1} + N_{a2}}$$

$$F = \frac{G_1 G_2 N_s + N_d + G_2 N_{a1} + N_{a2}}{G_1 G_2 (N_s + N_d)}$$

$$\Rightarrow \quad F - 1 = F_1 - 1 + \frac{F_2 - 1}{G_1}$$

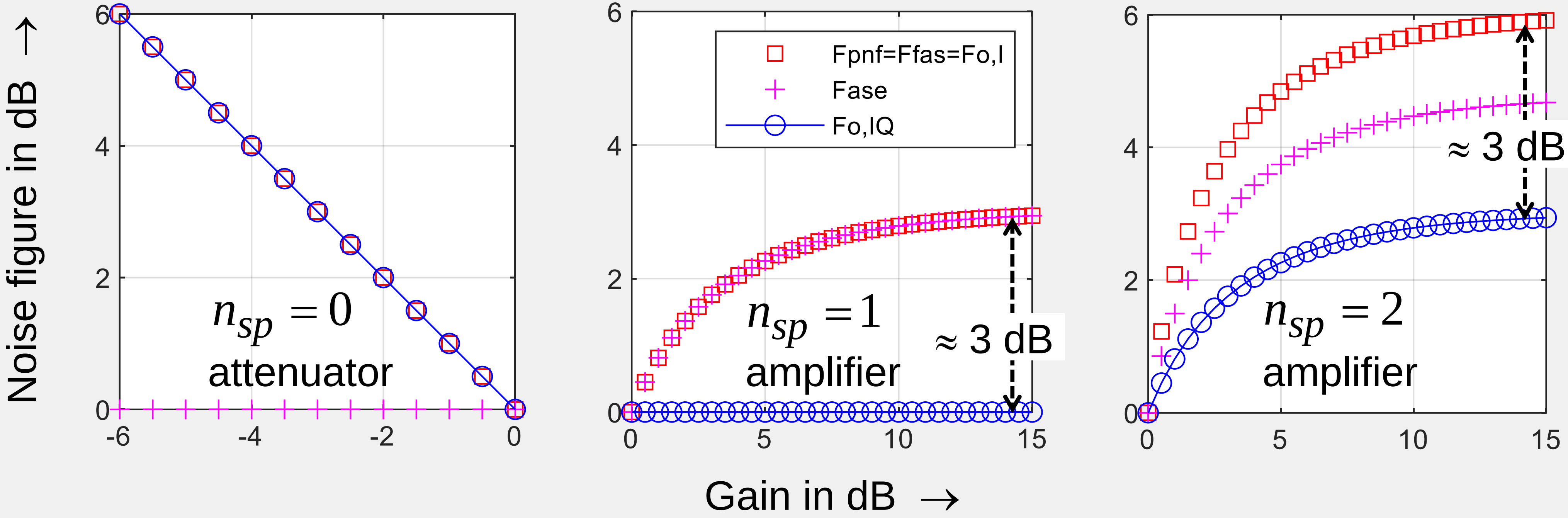
Complete induction yields Friis’ formula:

$$F - 1 = \sum_{i=1}^{n} \frac{F_i - 1}{\prod_{k=1}^{i-1} G_k}$$

It holds for all noise figures which can be written like this,

including $\underset{A=1}{F_e, \; F_{ase}}$, $\underset{A=0}{F_{pnf} = F_{fas} = F_{o,I}, \; F_{o,IQ}}$ and later $\underset{A=0\ldots1}{F_{IQ}, \; F_I}$!

Optical noise figures [dB] vs. gain [dB]



Only $F_{o,IQ}$ behaves like F_e (1 for amplifier with $n_{sp} = 1$; $1/G$ for attenuator; linear).

2 RX quadratures, like F_e ☺

$$F_{o,IQ} = n_{sp}\left(1-1/G\right)+1/G$$

Nonlinear F_{pnf}; 1 RX quadrature ϟ

$$F_{pnf} = 2n_{sp}\left(1-1/G\right)+1/G = F_{fas} = F_{o,I}$$

Is not the SNR degradation factor in any optical RX. ⇒ Is no optical NF.

$$F_{ase} = n_{sp}\left(1-1/G\right)+1$$

Assumes source noise ϟ

Properties of noise figures

Type of noise figure F	SNR degradation factor	Linear	Available RX quadratures	F of ideal ampl. $G \rightarrow \propto$	F of atten., $G<1$	M of ampl.	Input-referred energy per mode, kT_{ex} or $\tilde{\mu}hf$
F_e	yes	yes	2	1	$1/G$	≥ 0	$kT(F-1)$
$F_{o,IQ} = n_{sp}(1-1/G)+1/G$	yes	yes	2	1	$1/G$	$n_{sp}-1 \geq 0$	$hf(F-1/G)$
$F_{pnf} = F_{fas} = F_{o,I}$ $= 2n_{sp}(1-1/G)+1/G$	yes	not F_{pnf}	1	2	$1/G$	$2n_{sp}-1$ ≥ 1	$hf(F-1/G)/2$
$F_{ase} = 1+n_{sp}(1-1/G)$	no	yes	2	2	1	$n_{sp} \geq 1$	$hf(F-1)$

Only $F_{o,IQ}$ matches conceptually with F_e !

F_{pnf}, F_{fas}, F_{ase} are contradicted by F_e definition!

Note: NF is lab jargon. Precisely, F is the noise factor and $(10\text{ dB})\cdot\log_{10}(F)$ is the noise figure.

For lowest NF of a cascade, order amplifiers according to ascending noise measure M.

$$M = \frac{F-1}{1-1/G}$$

Ideal optical amplifier noise figure at large gain is ... ?

$$\text{ideal optical NF} = \frac{\text{number of available quadratures in amplifier}}{\text{number of available quadratures in receiver}}$$

(according to the foregoing)

quadratures	1	2
optical amplifier	phase-sensitive	phase-insensitive
optical receiver	direct detection or homodyne	I&Q, or heterodyne with image rejection

Common answer since mid 1990ies:

$$F_{pnf} = 2 = F_{fas} = F_{o,I}$$

But with the same logic one could answer: $F_{o,IQ} = 1/2$

Other cases are considered as special.

It makes most sense to consider as normal the pairing of amplifiers and receivers with same number of available quadratures:

optical amplifier	phase-sensitive	phase-insensitive
optical receiver	homodyne or direct detection	I&Q, or heterodyne with image rejection
Nonlinear! Does not yield a NF!	$F_{o,I} = 1$	$F_{o,IQ} = 1$ (like $F_e = 1$)

Standard! By far most frequent optical and electrical scenario today!

User must provide phase reference! RX can also contain phase-sensitive amplifier!

1 NF per detector type? 1 NF for 2 unequal scenarios?

One cannot say one NF (F_e) is for electrical detectors and another (F_{pnf}) is for quantum detectors (photodiodes), because valid NF definition prescribes power measurements but not how powers are measured. NF must be detector-independent! "Noise figure" without additions suggests the properties of F_e, i.e. SNR degradation factor in linear system with 2 RX quadratures (and preferably Gaussian noise).

$\Rightarrow$ Term "optical noise figure" seems fit only for $F_{o,IQ} = \tilde{\mu} + 1/G$.

To avoid misinterpretation, $F_{pnf} = 2\tilde{\mu} + 1/G$ could be called "high-power optical χ^2 (chi-square) noise estimator", "photoelectron number fluctuation indicator", ...

Likewise, $F_{o,I}$ $(= F_{fas} (= F_{pnf}))$ can be called "optical 1-quadrature/homodyne NF". When SNR is defined with only in-phase noise then the electrical 1-quadrature NF $F_{e,I}$ equals F_e. I have combined $F_{e,I}$ with $F_{o,I}$ to form a 1-quadrature NF F_I.
Result is similar to a corrected F_{fas}. But number of quadratures in F_{fas} is not given and one is left to assume that in the electrical domain F_{fas} is for 2 quadratures. $1 \neq 2$!
An interpretation difference is that in F_{fas} added thermal noise is considered not separately, but as caused by spontaneous emission (set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$). In a phase-sensitive amplifier, ideal $F_{o,I} = F_{fas} = 1$.

Removing avoidable receiver or power meter noise

In NF measurement the power meter or RX is always assumed to be free of avoidable noise. In practice it is not possible to cool a power meter or RX to $0\ \mathrm{K}$ in order to avoid its thermal noise. For this reason the intrinsic power meter noise is measured, and subtracted during NF measurement, thereby maximizing the resulting NF.

In the coherent RX we also must assume zero thermal noise. In the foregoing this has been achieved by letting $P_{LO} \to \infty$. Practically one must subtract RX thermal noise.

Shot noise of LO is unavoidable. But shot noise of received signal is avoidable by $P_{LO} \to \infty$. Practically one must subtract shot noises caused by GP_S and P_S .

Nonideal quantum efficiency η also reduces and falsifies measured SNR degradation. Hence we have assumed $\eta = 1$ in the responsivity $R = \eta e/(hf)$. Practically one must correct measurements such that they represent the case $\eta = 1$.

In the coherent I&Q RX the signal splitter can be viewed as a 2×2 coupler. When considering all frequencies, thermal noise enters also at the 2nd, unused coupler input. That can be avoided by cooling the termination of 2nd coupler input to $0\ \mathrm{K}$. Practically, thermal noise due to 2nd coupler input must be subtracted.

See implementation (pp. 50-51) of these corrections. – But measuring powers directly (p. 34) is easier! Yet also this duly minimizes RX noise and maximizes measured NF.

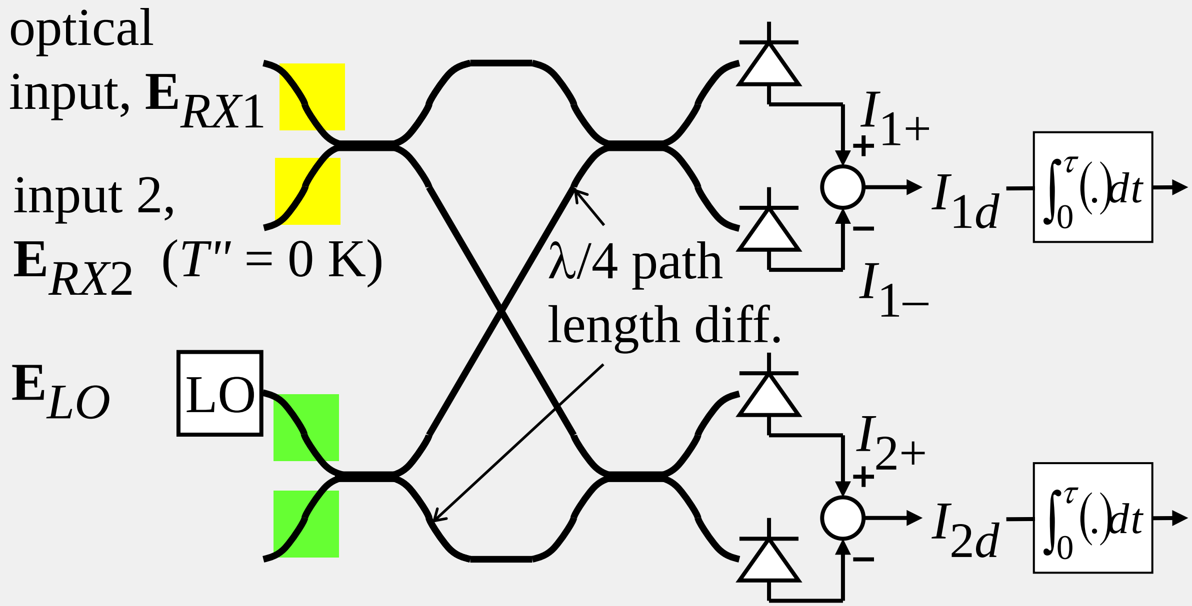
Thermal and optical noise

Photocurrent shot noise is explained either semiclassically by a Poisson distribution of photoelectrons, or by zero point fluctuations of fields at all inputs.

Thermal noise energy per mode approaches kT only at low frequencies f. For all f the correct expression is:

$$\frac{hf}{e^{hf/(kT)}-1}$$

In NF measurement, an ideal RX (or power meter) is assumed! $\Rightarrow$



For the source with temperature T we define:

$$k'T=\frac{hf}{e^{hf/(kT)}-1}$$

Signal input 2 shall be terminated by an absorber having $T''=0\text{ K}$. No thermal noise enters there.

All noises interfere essentially with the strong LO fields ($P_{LO}\rightarrow\infty$).

LO interferences of noises from the signal inputs add / LO inputs cancel upon photocurrent subtractions.

Total intrinsic (source+detection) noise energy per mode

Only 1/2 of these energies per arriving mode mainfests per received quadrature!

Thermal and added amplifier and detection shot noise can be measured as fluctuations (AC components) in RX.

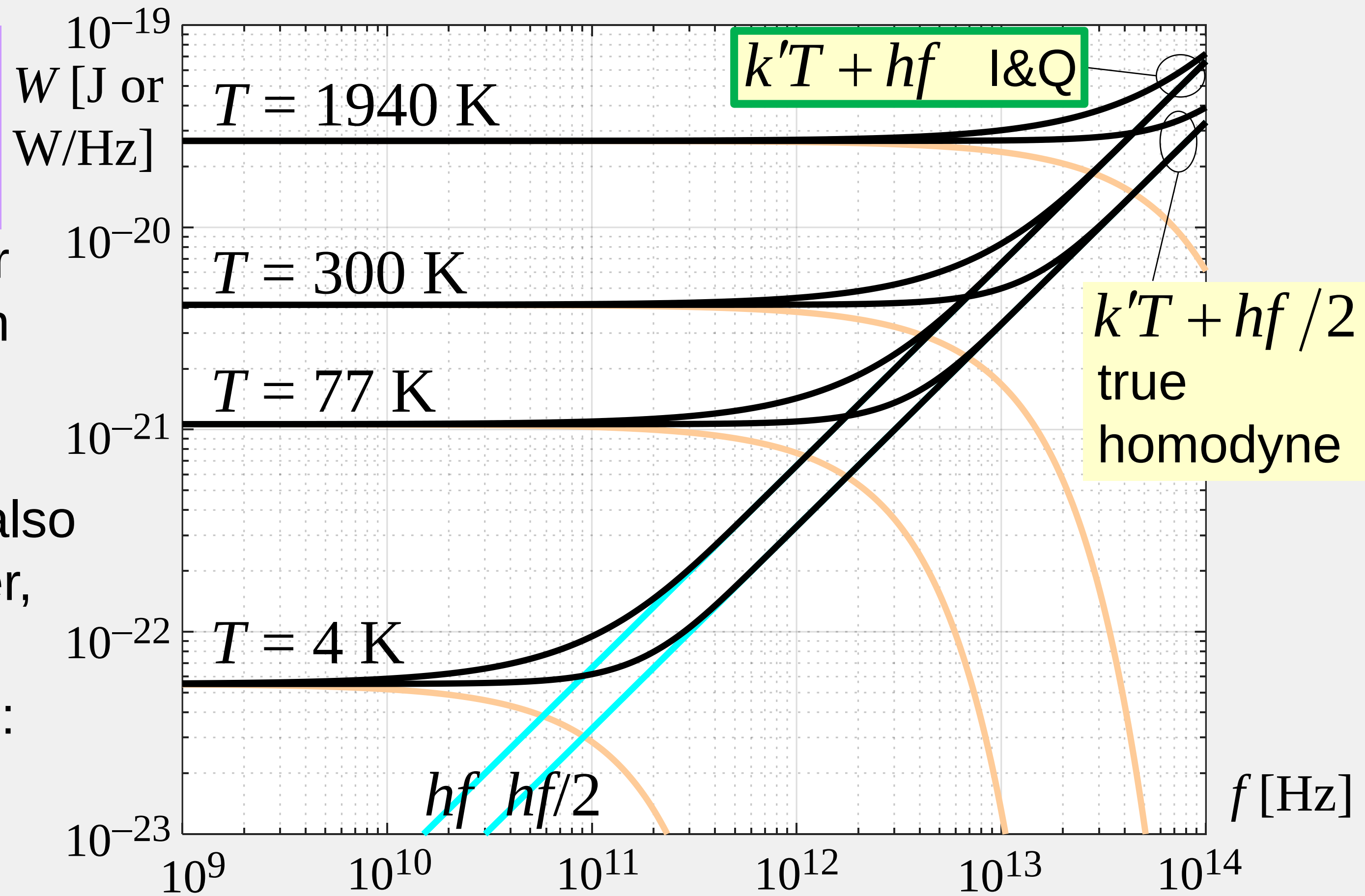
Except shot noise, this can also be measured as mean power, using a simple power meter.

Shot noise energy per mode:

hf in RX for 1 mode or 2 quadratures

$hf/2$ in RX for 1/2 mode or 1 quadrature

B.M. Oliver, "Thermal and Quantum Noise", Proceedings of the IEEE, 1965, pp. 436-454



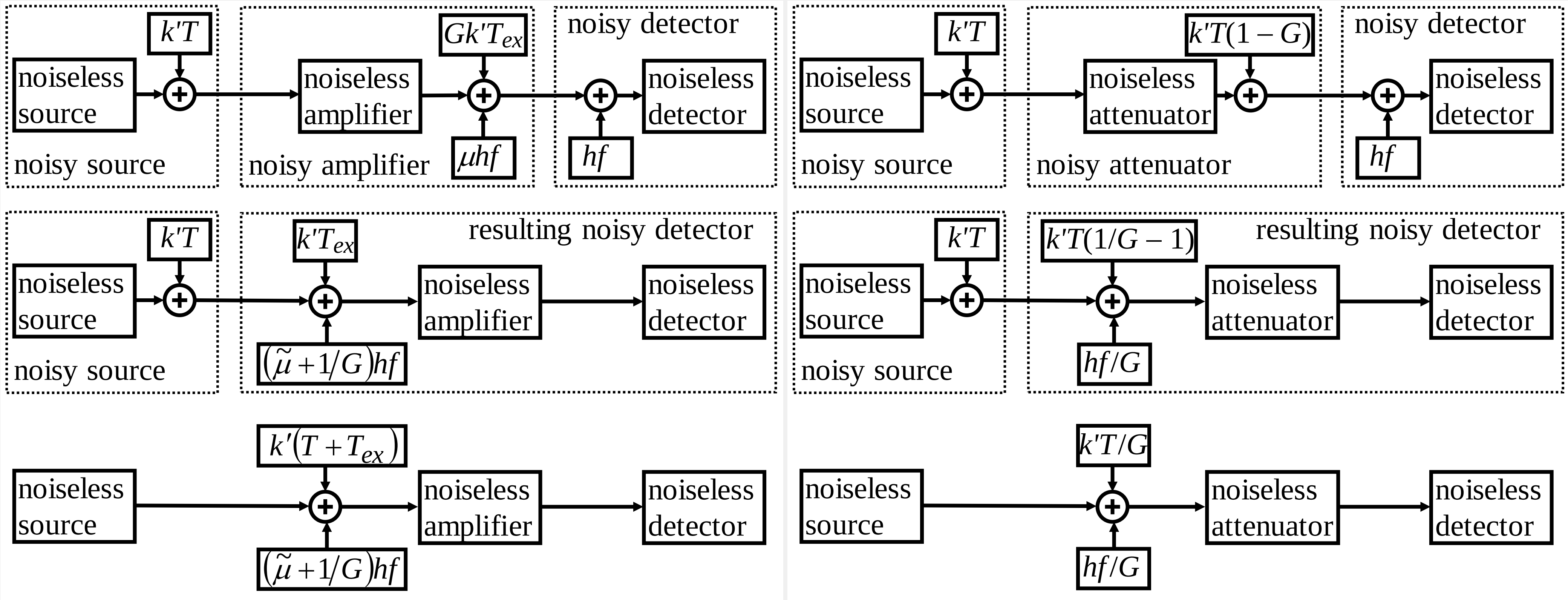
Shot noise or zero-point fluctuations, not measurable with mean power meter

Thermal noise, measurable with mean power meter

$$k'T = \frac{hf}{e^{hf/(kT)} - 1}$$

Univ. Paderborn, R. Noe

Block diagrams with thermal and optical noise



Source, amplifier (left) or attenuator (right) and detector (electrical or coherent optical), **all I&Q**, noisy or noiseless with equivalent added noise energies per mode. Individual devices (top) and equivalent interpretations (middle, bottom).

If detector were for only 1 available quadrature, detection noise hf would become $hf/2$.

Upconversion e-o is possible with an I&Q modulator (or DSB modulator + SSB filter), downconversion o-e with an I&Q RX (or heterodyne + image rejection filter).

SNR in the presence of thermal and optical noise

To derive a consistent unified NF (**I&Q** !) we add noises of F_e and $F_{o,IQ}$ for all f.

Optical and electrical gains G are identical because they manifest at same f.

Thermal noise in bandwidth $B_o = 1/\tau = 2B_e$ is $GF_e k'TB_o$ at amplifier output.

Half of this is in phase with signal. In coherent I&Q RX it appears multiplied with $R^2 P_{LO}$, like amplified signal power GP_S. Corresponding variance σ_e^2 is added.

$$SNR_{IQ,out} = \frac{\overline{I_{1d}}^2}{\sigma_e^2 + \sigma_{I_{1d}}^2 + \sigma_{I_{1s}}^2}$$

Detector type does not matter, as long as it is usable in linear I&Q receiver:

$$= \frac{R^2 P_{LO} G P_S}{R^2 P_{LO} G F_e k'TB_o/2 + R^2 P_{LO}\tilde{\mu} G h f B_o/2 + eRP_{LO}B_e}$$

Powers in I&Q receiver with quantum detectors

$$= \frac{GP_S}{GF_e k'TB_o/2 + \tilde{\mu} G h f B_o/2 + hfB_e}$$

Powers in electrical I&Q receiver

$$= \frac{P_S\tau}{F_e k'T/2 + F_{o,IQ}hf/2} = \frac{P_S\tau}{k'(T + T_{ex})/2 + (\tilde{\mu} + 1/G)hf/2}$$

- Thermal source noise
- Thermal amplifier noise
- Spontaneous emission field noise in amplifier
- Shot noise in detector

I&Q noise figure from electrical to optical frequencies

$$SNR_{IQ,out} = \frac{P_S \tau}{F_e k'T/2 + F_{o,IQ} hf/2} = \frac{P_S \tau}{k'(T+T_{ex})/2 + (\tilde{\mu} + 1/G)hf/2}$$

Signal energy

Intrinsic I&Q noise energy per received quadrature

$$SNR_{IQ,in} = \frac{P_S \tau}{k'T/2 + hf/2}$$ (obtained with $T_{ex} = 0,\ \tilde{\mu} = 0,\ G = 1$)

Pure Gaussian noises! Linear! 2 available RX quadratures!

$$\frac{SNR_{IQ,in}}{SNR_{IQ,out}} = F_{IQ} = \frac{F_e k'T + F_{o,IQ} hf}{k'T + hf} = \frac{k'(T+T_{ex}) + (\tilde{\mu} + 1/G)hf}{k'T + hf}$$

$$= A + (1-A)/G + (AT_{ex}/T + (1-A)\tilde{\mu})$$

$$A = k'T/(k'T + hf)$$

Fulfills Friis' formula!

Measured F_{IQ} is just observed SNR degradation in linear system with 2 quadratures.

In amplifier, F_e, $F_{o,IQ}$ may not be known. Anyway, $k'T_{ex} + \tilde{\mu} hf$ is total added noise.

In attenuator, clear separation yields the correct result: $G < 1$, $T_{ex} = T(1/G - 1)$,

$n_{sp} = 0,\ \tilde{\mu} = 0 \quad \Rightarrow \quad F_{IQ} = 1/G = F_e = F_{o,IQ}$

At low f: $F_{IQ} \to F_e$. At high f: $F_{IQ} \to F_{o,IQ}$.

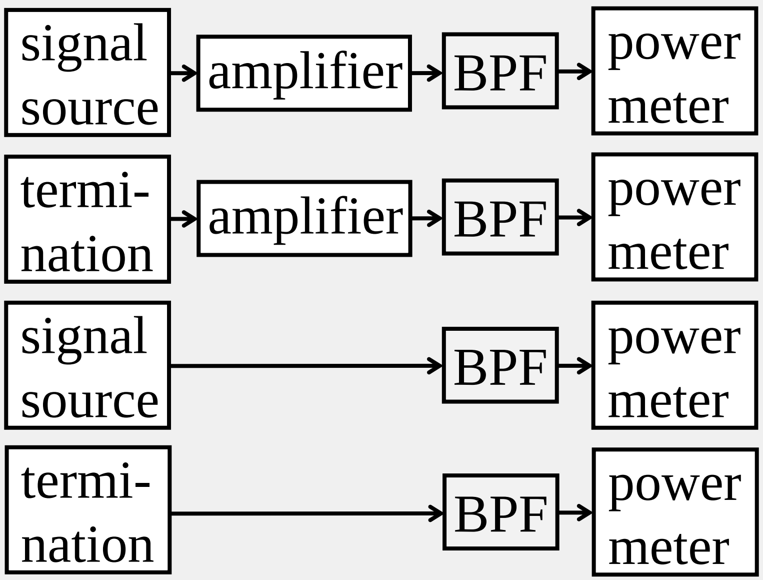
https://ieeexplore.ieee.org/document/9783564 = 66 GHz @ 4 K

At 13400 / 1940 / 300 / 77 / 4 K, equal $k'T = hf$ is at f = 194 / 28 / 4.3 / 1.1 / 0.06 THz.

Measure I&Q noise figure with power meter

Usually there are $p=2$ polarization modes. $p=1$ needs polarizer to be inserted.

P_0' is offset due to noise generated outside B_o and inside power meter. Source and added noises $k'T + k'T_{ex} + \tilde{\mu} hf$ are detected as static power. Not so shot noise hf/G.



$$P_1 = p\left(Gk'\left(T+T_{ex}\right)+G\tilde{\mu}hf\right)B_o + P_0' + GP_S$$

$$P_2 = p\left(Gk'\left(T+T_{ex}\right)+G\tilde{\mu}hf\right)B_o + P_0'$$

$$P_3 = pk'TB_o + P_0' + P_S$$

$$\left(k'T = \frac{hf}{e^{hf/(kT)}-1}\right)$$

$$P_4 = pk'TB_o + P_0'$$

$$G = \frac{P_1 - P_2}{P_3 - P_4}$$ Gain

$$k'T_{ex} + \tilde{\mu}hf = \frac{1}{G}\left(\frac{P_2 - P_4}{pB_o} - (G-1)k'T\right)$$ Added noise

It doesn't matter, and needn't be known, in how far added noise is of thermal or quantum origin. F_{IQ} and all other NF can be determined from simple static power measurements.

$$F_{IQ} = \frac{k'T + \left(k'T_{ex} + \tilde{\mu}hf\right) + hf/G}{k'T + hf}$$

SNR with 1-quadrature noise and homodyne receiver

No power splitting ⇒ $P_{LO}, P_S, P_n, \tilde{\mu}, n_{sp}$ must be multiplied by **2** compared to $F_{o,IQ}$ calculation. Only 1 RX input! Total thermal noise in bandwidth B_o at amplifier output is $GF_e k'TB_o$. Half of this is in phase with the signal. In the coherent 1-quadrature (homodyne) RX it appears multiplied with $4R^2 P_{LO}$, like the amplified signal power GP_S. RX for 1 quadrature is a special case!

$$SNR_{I,out} = \frac{4\overline{I_{1d}}^2}{4\sigma_e^2 + 4\sigma_{I_{1d}}^2 + 2\sigma_{I_{1s}}^2}$$

(Quantities found in I&Q RX are multiplied here by **2**·**2** or **2**.)

$$F_{e,I} = F_e$$

$$= \frac{4R^2 P_{LO} G P_S}{4R^2 P_{LO} GF_e k'TB_o/2 + 4R^2 P_{LO}\tilde{\mu} GhfB_o/2 + 2eRP_{LO}B_e}$$

Powers in homodyne receiver with quantum detectors

$$= \frac{2GP_S}{2GF_e k'TB_o/2 + 2\tilde{\mu} GhfB_o/2 + hfB_e}$$

$$= \frac{2P_S\tau}{F_e k'T + F_{o,I} hf/2} = \frac{2P_S\tau}{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}$$

Thermal source noise
Thermal amplifier noise
Spontaneous emission field noise in amplifier
Shot noise in detector

1-quadrature / homodyne unified noise figure

$$SNR_{I,out} = \frac{2P_S\tau}{F_e k'T + F_{o,I} hf/2} = \frac{2P_S\tau}{k'(T+T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}$$

Signal energy

Intrinsic homodyne noise energy in received quadrature

$$SNR_{I,in} = \frac{2P_S\tau}{k'T + hf/2} = \frac{P_S\tau}{k'T/2 + hf/4} \quad \text{(for } T_{ex}=0,\ \tilde{\mu}=0,\ G=1\text{)}$$

$$\frac{SNR_{I,in}}{SNR_{I,out}} = F_I = \frac{F_e k'T + F_{o,I}\, hf/2}{k'T + hf/2} = \frac{k'(T+T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}{k'T + hf/2}$$

$$= A_I + (1-A_I)/G + (A_I T_{ex}/T + (1-A_I)2\tilde{\mu}) \qquad A_I = k'T/(k'T + hf/2) \neq A$$

$F_{o,I} \neq F_{o,IQ}$ because there is detection noise!

$F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!

(set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$)

1-quadrature / homodyne F_I is close to F_{fas} (except k' and interpretation difference)!

In definition of F_{fas}, number of quadratures was not discussed. F_{fas} is intended to be identical with the normal electrical F_e, which is understood to be for 2 available RX quadratures. One is left to assume that F_{fas} is for 2 available RX quadratures in electrical, 2…1 in thermal (low THz) and 1 in optical domain. That is contradictory!

1-quadrature / homodyne unified noise figure

Electrical part of F_I is expected to be of minor importance.
Reason: Due to thermal source noise and absence of detection shot noise, electrical homodyne does not have an SNR advantage over electrical I&Q (when evaluating only 1 quadrature).

$$\frac{SNR_{I,in}}{SNR_{I,out}} = F_I = \frac{F_e k'T + F_{o,I}\, hf/2}{k'T + hf/2} = \frac{k'(T+T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}{k'T + hf/2}$$

$$= A_I + (1-A_I)/G + (A_I T_{ex}/T + (1-A_I)2\tilde{\mu})$$

$$A_I = k'T/(k'T + hf/2) \neq A$$

$F_{o,I} \neq F_{o,IQ}$ because there is detection noise!

$F_{e,I} = F_{e,IQ} \equiv F_e$ because there is source noise!

(set $T_{ex} = 0$ and take a high $\tilde{\mu}$, with $\tilde{\mu} \to \infty$ for $f \to 0$)

1-quadrature / homodyne F_I is close to F_{fas} (except k' and interpretation difference)!

Attenuator: I simply say

$T_{ex} = T(1/G - 1)$, $n_{sp} = 0 = \tilde{\mu}$,

$F_I = 1/G = F_{o,I} = F_e \quad (= F_{e,I})$.

Attenuator: To get $F_{fas} = 1/G \quad (= F_I)$ I find I must set $T_{ex} = 0$, $n_{sp} = -k'T/(hf)$, $\tilde{\mu} = n_{sp}(1 - 1/G)$.

$f \to \{\infty, 0\} \Rightarrow n_{sp} \to \{0, -\infty\}$, $\tilde{\mu} \to \{0, \infty\}$!

Noise figures

RX or power detector noise would cause F_e to be underestimated. Therefore RX noise is always subtracted, using reference measurements. The same way, avoidable optical RX noise can and must be subtracted at high f. Only LO shot noise is fundamental and is kept. Photodiode efficiency must be set equal to 1. We get:

Electrical $kT >> hf$	Unified/generalized		Optical $kT << hf$
kT	$k'T = \frac{hf}{e^{hf/(kT)}-1}$	$\tilde{\mu} = n_{sp}(1-1/G)$	$\tilde{\mu} = n_{sp}(1-1/G)$

RX with 2 available quadratures (I&Q), i.e. 1 mode; F_{IQ} is the noise figure:

$F_e \equiv F_{e,IQ} = 1 + \frac{T_{ex}}{T}$	$F_{IQ} = \frac{k'(T+T_{ex}) + (\tilde{\mu}+1/G)hf}{k'T+hf}$	$F_{o,IQ} = \tilde{\mu} + 1/G$

RX with 1 available quadrature; optical homodyne (and IM/DD without sp-sp):

$F_{e,I} = 1 + \frac{T_{ex}}{T}$	$F_I = \frac{k'(T+T_{ex}) + (2\tilde{\mu}+1/G)hf/2}{k'T+hf/2}$	$F_{o,I} = 2\tilde{\mu} + 1/G$

Shot noise occurs upon detection.

$\Rightarrow$ Optical homodyne is different.

In all cases, NF of pure attenuator is $1/G$.

Summary

- All prior optical and unified NF F_{pnf}, F_{fas}, F_{ase} are in conflict with electrical NF F_e.
- A „noise figure“ without special name is expected to be the SNR degradation factor in a linear system with 2 available quadratures (and Gaussian noise?!), like F_e.
- The only optical NF which fulfills this is the optical I&Q NF $F_{o,IQ}$. It is ≥ 1, like F_e.
- Coherent I&Q receivers are linear field sensors. They linearize the quadratic field behavior of photodiodes. Heterodyne with image rejection is also fine.
- At high gain, $F_{o,IQ} \approx F_{pnf} / 2$, i.e. ≈ 3 dB less when expressed in dB.
- Electrical F_e and optical $F_{o,IQ}$ are limit cases of the NF F_{IQ}, unified for all f.

  Quantum noise / F_{IQ} plays a role in today‘s RF electronics at low $T = 4$ K.
- The in-phase equivalent of $F_{o,IQ}$ is $F_{o,I}$, a limit case of the unified F_{fas}. So, F_{fas} is a 1-quadrature NF and its other limit is F_e for 1 quadrature, not the assumed 2.
- Information conveyed by the full F_{pnf} (including sp-sp) of a specific DD RX can be obtained, more accurately, from $F_{o,IQ}$ (pure Gaussian noise).

  (after correction $k \rightarrow k'$)
- Optical amplifier adds Gaussian I&Q field noise (wave aspect). Photodetection adds shot noise (particle aspect). (See support material on p. 1.)

Quantities needed for electrical noise figure calculation

G	(available) power gain of device
$P_{s,in}$	input signal power
$P_{s,out} = GP_{s,in}$	output signal power
$B = 1/\tau$	one-sided, physical bandwidth around a carrier frequency
τ	observation or integration time, length of BPF response
$P_{n,in} = kTB = kT/\tau$	(thermal) input noise power
T	source temperature
kT	thermal noise energy in 1 mode, having 2 quadratures (cos, sin)
$kT/2$	thermal noise energy in 1 quadrature; see kT/C noise where any noisy conductance $G \geq 0$ causes a voltage variance $U^2 = kT/C$ at capacitance C, and $(C/2)U^2 = kT/2$ is mean stored energy.
$P_{n,out}$	output noise power
SNR_{in}	input signal-to-noise ratio
SNR_{out}	output signal-to-noise ratio
F	noise factor, often called noise figure (lab jargon)

Definition of electrical noise figure

$$SNR_{in} = \frac{P_{s,in}}{P_{n,in}/2}$$

input signal-to-noise ratio.

$$SNR_{out} = \frac{P_{s,out}}{P_{n,out}/2} = \frac{GP_{s,in}}{P_{n,out}/2}$$

output signal-to-noise ratio

Noise and gain are available in 2 quadratures with equal strengths!

For F it does not matter whether noise power in 1 mode is divided by 2, like above. If so, then this is the noise in 1 of 2 available RX quadratures of the mode. F always assumes 2 available RX quadratures! – One may take energies instead of powers.

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{n,out}}{GP_{n,in}}$$

noise figure ≥ 1

$F_{ex} = F - 1$ excess noise factor

$T_{ex} = (F-1)T$ excess noise temperature

$$F - 1 = \sum_{i=1}^{n} \frac{F_i - 1}{\prod_{k=1}^{i-1} G_k}$$

Friis‘ formula

Noise added at amplifier output is divided by gain to represent noise that must be added at input of a noiseless amplifier. Contributed noise powers in an amplifier chain, each divided by total prior gain, may be added at input. Available gains in amplifier chain are multiplicative.

Electrical noise figures of amplifier and attenuator

$$F_e = \frac{P_{n,out}}{GP_{n,in}} = \frac{P_{n,out}}{GkTB} = \frac{Gk(T+T_{ex})B}{GkTB} = 1 + \frac{T_{ex}}{T}$$ NF of electrical amplifier

$$P_{n,out} = kTB$$ in electrical attenuator (works as noisy resistor)

$$F_e = \frac{1}{G} \geq 1$$ NF of electrical attenuator ($G < 1$)

$$T_{ex} = T(1/G - 1)$$ excess noise temperature of attenuator

---

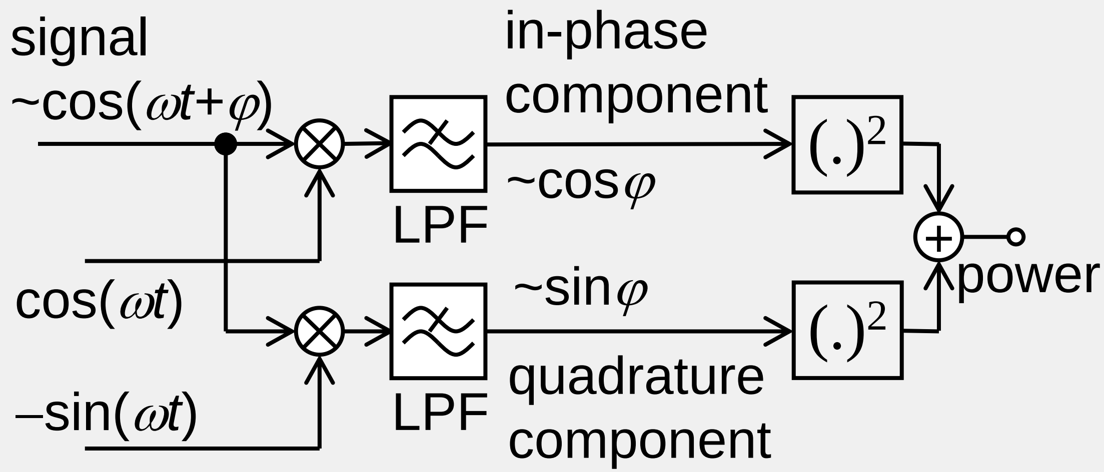
Optionally, phase-sensitive detection allows measuring power in only 1 quadrature. Noise power is thereby halved if it comes from a source (such as a resistor or normal amplifier) which generates noise in 2 quadratures. Normal electrical amplifiers pass 2 available quadratures. RX = detector also has 2 available quadratures. But powers (signal, thermal noise) are measurable also with asynchronous power detectors (thermal; Schottky diode). 2 available quadratures are implied for F_e measurement. This means: A unified all-frequency NF which has the normal electrical NF F_e as its electrical limit case must have 2 available RX quadratures like F_e, at all frequencies.

How to measure electrical powers

Asynchronous electrical power measurement in the carrier or radio frequency domain yields total power in 2 quadratures. This is also possible via downconversion:

2 quadratures are always available.

Using 2 mixers with local oscillator signals having $\pi/2$ mutual phase difference, one downconverts to the baseband. Addition of the squares of the resulting I&Q signals gives total power in the 2 quadratures.



If there is only 1 mixer to the baseband then noise is measured in only 1 quadrature.

If a downconversion receiver is a heterodyne receiver it needs an image rejection filter at the input. Otherwise doubled noise power from the amplifier would be measured (for a given input noise power spectral density).

NF measurement is transparent to frequency up/downconversion.

NF is SNR degradation factor; 2 available RX quadratures; linearity; powers ~ squares of amplitudes: All this must hold for all frequencies! Hopefully also: Gaussian noise; ideal NF = 1

Gain and noise energy in optical amplifiers

$dP/dt = aP$	power evolution in purely amplifying optical medium
$dP/dt = -bP$	power evolution in purely attenuating optical medium
$G = e^{(a-b)t} = e^{(a-b)z/v_g}$	power gain of optical amplifier (group velocity: v_g)
$P_n\tau = \mu hf = G\tilde{\mu}hf = n_{sp}(G-1)hf$	added output noise energy per mode = noise energy in independent I&Q carrier sample = J / (s (observation time) · Hz (bandwidth))
$n_{sp} = \dfrac{a}{a-b}$	spontaneous emission factor, calculated from stimulated emission a and absorption b coefficients per time unit
μ at output or $\tilde{\mu}$ at input	expectation value of equivalent noise photons per mode

Wave aspect: During electromagnetic wave propagation, no photons manifest!

Particle aspect: Generation and detection of light occurs with quantized energy and can be explained by photons, not waves.

See pp. 52-61. Photons in this talk manifest only during their absorption or emission! For waves it is the opposite, they exist only during transmission.

Noise figure F_{fas}

"*fas*" = "fluctuation of amplitude squares"

Fluctuation of amplitudes: SNR = (squared mean) / variance

Fluctuation of amplitude squares: SNR = (squared mean of the square) / (variance of the square). That's exactly what happens for F_{pnf}! Quadratic nonlinearity!

But F_{fas} is determined by "linear" detection. $\Rightarrow$

Name "fluctuation of amplitude squares" is misleading compared to "linear" detection.

Expressions for F_{fas}:

$$F_{\text{fas}} = \frac{\text{SNR at input}}{\text{SNR at output}} = \frac{\dfrac{h\nu\langle n_s\rangle}{h\nu/2}}{\dfrac{h\nu G\langle n_s\rangle}{(h\nu/2)\{1+\theta(G-1)\}}} = 1+\theta(1-1/G). \quad (13)$$

(earlier versions)

$\theta \equiv n_{sp}$

$\theta(1-1/G) \equiv \tilde{\mu}$

2...3 unequal, conflicting versions of F_{fas}!

$\Rightarrow F_{fas} = \tilde{\mu} + 1/G = F_{o,IQ}$ I&Q NF?

Probably not, since this seems to be a mistake, instead of G.

$\Rightarrow F_{fas} = \tilde{\mu} + 1 = F_{ase}$ Not an optical NF!

$(1/2)G + \frac{1}{2}\theta(G-1) \Rightarrow F_{\text{fas}} = F_{\text{ASE}}\,(15) \Rightarrow$

(final version)

$$F_{\text{fas}} = \frac{\text{SNR}_i}{\text{SNR}_o} = 1 + \frac{(2\theta-1)\left(1-\dfrac{1}{G}\right)}{2\langle n_\theta\rangle + 1}. \quad (18)$$

$\langle n_\theta \rangle = 0$ Needs 1-quadrature (homodyne) receiver, or direct detection!

$\Rightarrow F_{fas} = 2\tilde{\mu} + 1/G = F_{o,I} = F_{pnf}$

- $F_{pnf} = 2\tilde{\mu} + 1/G$ involves nonlinearity and requires redefinition of power. For that reason it is not a NF. And it maps 2 quadratures into 1.
- $F_{\mathrm{fas}} = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o} = 1 + \frac{(2\theta - 1)\left(1 - \frac{1}{G}\right)}{2\langle n_\theta \rangle + 1}$ becomes $F_{fas} = 2\tilde{\mu} + 1/G = F_{o,I}$ in the optical domain, which means there is 1 quadrature. It is defined for all f with the intention of becoming $F_{fas} = F_e$ in the electrical domain, where there are 2 quadratures. But number of quadratures must not change vs. f. Hence F_{fas} is not a unified NF, due to $1 \neq 2$. If 1 RX quadrature had been specified for optical and electrical, and k corrected into k', F_{fas} would be the 1-quadrature unified NF, due to $F_{e,I} = F_e$.
- Exactly this has been specified for the 1-quadrature unified NF

$$F_I = \frac{k'(T + T_{ex}) + (2\tilde{\mu} + 1/G)hf/2}{k'T + hf/2}.$$

(F_{fas} is close to this special case.)

- A NF without special denomination must correspond to F_e with its 2 RX quadratures. Therefore the optical NF is only $F_{o,IQ} = \tilde{\mu} + 1/G$ and the unified NF is only

$$F_{IQ} = \frac{k'(T + T_{ex}) + (\tilde{\mu} + 1/G)hf}{k'T + hf}.$$

2 RX quadratures are common today in cases where amplifier noise matters!

- $F_{ase} = 1 + \tilde{\mu}$ is not the SNR degradation factor in any optical receiver. So, F_{ase} is not an optical NF. Hence it is pointless to generalize it to a unified F_{ase}.

Traditional optical noise figure F_{pnf}

$SNR_{pnf} = \langle n \rangle^2 / \sigma_n^2$

Defined by optical amplifier pioneer E. Desurvire (1990ies), denoted *p*hoton *n*umber *f*luctations by H.A. Haus (1998).

Photon number n ~ photocurrent ~ optical power $P \sim E \times H \sim |\mathbf{E}|^2$. This means:

Amplitude squarer is „built“ into signal path, which becomes nonlinear!

„Power“ is ~ $|\mathbf{E}|^4$! F_{pnf} with sp-sp beat noise depends on power+bandwidths!

2 optical quadratures → only 1 available electrical quadrature!

Minimum NF of amplifier with large gain: $F_{pnf} = 2$

Widely used! Defined for optical DD receiver. But today's standard in applications where amplifier noise matters are coherent optical I&Q RX.

A check: If photons were present in the wave then this would hold also at the input of a ficticious noise-free amplifier with $n_{sp} = 0$. According to the below that would improve SNR by a factor G. This cannot be.

neglect for $\langle n \rangle \to \propto$

$$SNR_{pnf,in} = \frac{\langle n \rangle^2}{\langle n \rangle}$$ for Poisson distribution

$$SNR_{pnf,out} = \frac{G^2 \langle n \rangle^2}{G\langle n \rangle + n_{sp}\left(2(G-1)G\langle n \rangle + \ldots\right)}$$

$$F_{pnf} = \frac{SNR_{pnf,in}}{SNR_{pnf,out}} \approx \frac{1 + 2n_{sp}(G-1)}{G} = 2\tilde{\mu} + 1/G$$

signal shot noise | signal-sp(ontaneous) beat noise | sp-sp beat noise

Optical noise figure F_{ase}

$$F_{ase} = 1 + \tilde{\mu}$$

Defined and denoted amplified spontaneous emission by noise figure pioneer H.A. Haus (1998).

Linear system, 2 quadratures, power ~ squared field amplitude, like for F_e!

But:

F_{ase} is not the SNR degradation factor in any optical receiver.

So, F_{ase} is not an optical NF.

$F_{ase} \geq 2$ for amplifiers with high gain

Due to NF confusion I have initially used in my lectures only $\tilde{\mu}$. After $F_{ase} = 1 + \tilde{\mu}$ was defined I have used it as optical NF, also in https://doi.org/10.1007/978-3-662-49623-7. But that doesn‘t make sense: It is not the SNR degradation factor.

H.A. Haus defined his NF as replacements of F_{pnf}, obviously because he did not accept F_{pnf} to be a valid NF. Seemingly, all NF definitions prior to $F_{o,IQ}$ have been guided by the idea that the NF of an ideal optical amplifier should be 2. But an optical NF without special denomination (such as homodyne) implies that there are 2 available RX quadratures, like in the electrical domain. That or the SNR degradation have not been complied with. In reality, an ideal optical amplifier does not degrade the SNR in a system with 2 available RX quadratures. It hence has an optical NF of 1 ($= F_{o,IQ}$).

Prior unified noise figures

H.A. Haus has standardized the electrical NF. Later (2000) he was the first to bring up inspiring unified NF. Using kT and hf noises he has generalized his two optical NF for all f from electrical, where they seemingly shall become F_e, to optical:

A NF F_{fas} (fluctuation of amplitude squares) was defined finally, and generalized. But:

For rising f (and depending on T; transition $f \approx kT/h$),

2 electrical quadratures must <u>gradually</u> become 1 optical quadrature,

signal detection <u>gradually</u> needs phase recovery. **(how?)**

If one does the same with F_{pnf} (for large input power) then, **for rising f,**

signal phase is <u>gradually</u> lost,

the linear system must <u>gradually</u> become nonlinear, **(how?)**

2nd must <u>gradually</u> become 4th powers of amplitudes.

F_{ase} was also generalized from electrical to optical f. But:

F_{ase} **is not the SNR degradation factor in any optical receiver. (no NF!)**

All above: kT **was taken for all f, which is correct only for $hf << kT$.**

⇒ **All prior optical and unified/generalized NF violate systematics of electrical NF or linearity requirement or technical function!**

Synchronous downconversion allows separating signal S and noise N. $B_o = 1/\tau = 2B_e$.
Responsivity $R = \eta e/(hf)$ is known. Electrical RX part has gain H and equivalent thermal noise PSD $d\langle i^2 \rangle / df$. At 2nd RX input, absorber temperature is T'' and thermal noise energy per mode is $k''T'' = hf / \left(e^{hf/(kT'')} - 1 \right)$. We can determine:

1. No signal, with amplifier

$$N_1 = H\left(R^2 P_{LO}(GF_e k'T + k''T'') + R^2 P_{LO} \tilde{\mu} Ghf + eRP_{LO} + d\langle i^2 \rangle / df \right) B_e$$

2. No signal, no amplifier

$$N_2 = H\left(R^2 P_{LO}(k'T + k''T'') + eRP_{LO} + d\langle i^2 \rangle / df \right) B_e$$

3. No signal, no amplifier, other source temperature

$$N_3 = H\left(R^2 P_{LO}(k'''T''' + k''T'') + eRP_{LO} + d\langle i^2 \rangle / df \right) B_e \qquad k'''T''' = hf / \left(e^{hf/(kT''')} - 1 \right)$$

4. Signal, amplifier $S_4 = HR^2 P_{LO} G P_S$ (Synchronous downconversion,

5. Signal, no amplifier $S_5 = HR^2 P_{LO} P_S$ to get signal without noise.)

From S_5 we get H. From S_4 and S_5 we get G. Without LO we get noise $H\, d\langle i^2 \rangle / df\, B_e$.

$$N_6 = \frac{N_2 k'''T''' - N_3 k'T}{k'''T''' - k'T} - H\eta eRP_{LO}B_e = H\left(R^2 P_{LO} k''T'' + e(1-\eta)RP_{LO} + d\langle i^2 \rangle / df \right) B_e$$

$$SNR_{out}=\frac{S_4}{N_1-N_6}=\frac{HR^2P_{LO}GP_S2\tau}{H\left(R^2P_{LO}GF_ek'T+R^2P_{LO}\tilde{\mu}Ghf+e\eta RP_{LO}\right)}=\frac{GP_S2\tau}{GF_ek'T+\tilde{\mu}Ghf+hf}$$

$$SNR_{in}=\frac{S_5}{N_2-N_6}=\frac{HR^2P_{LO}P_S2\tau}{H\left(R^2P_{LO}k'T+e\eta RP_{LO}\right)}=\frac{P_S2\tau}{k'T+hf}$$

$$F_{IQ}=\frac{SNR_{in}}{SNR_{out}}=\frac{GF_ek'T+G\tilde{\mu}hf+hf}{G(k'T+hf)}=\frac{k'(T+T_{ex})+(\tilde{\mu}+1/G)hf}{k'T+hf}$$

Alternative path, with the same result: Measure noises with signal

$$\hat{N}_1=H\left(R^2P_{LO}(GF_ek'T+k''T'')+R^2P_{LO}\tilde{\mu}Ghf+eR(P_{LO}+GP_S)+d\langle i^2\rangle/df\right)B_e$$

$$\hat{N}_2=H\left(R^2P_{LO}(k'T+k''T'')+eR(P_{LO}+P_S)+d\langle i^2\rangle/df\right)B_e$$

$$\hat{N}_3=H\left(R^2P_{LO}(k'''T'''+k''T'')+eR(P_{LO}+P_S)+d\langle i^2\rangle/df\right)B_e$$

$$N_7=\frac{\hat{N}_2k'''T'''-\hat{N}_3k'T}{k'''T'''-k'T}=H\left(R^2P_{LO}k''T''+eR(P_{LO}+P_S)+d\langle i^2\rangle/df\right)B_e$$

$$SNR_{out}=\frac{S_4}{\hat{N}_1-N_7+HeR\begin{pmatrix}(\eta-1)P_{LO}\\-GP_S\end{pmatrix}B_e}$$

$$SNR_{in}=\frac{S_5}{\hat{N}_2-N_7+HeR\begin{pmatrix}(\eta-1)P_{LO}\\-P_S\end{pmatrix}B_e}$$

Master equation of photo(electro)n statistics

- How many photons can be detected behind optical amplifiers / attenuators?
- Probability evolution of photon number ($dt \to 0$; multiple transitions neglected)

$$P(n,t+dt) = P(n|n)P(n,t) + P(n|n-1)P(n-1,t) + P(n|n+1)P(n+1,t)$$

stimulated emission, spontaneous emission, absorption

$$P(n|n-1) = ((n-1)a + c)dt \qquad P(n|n+1) = (n+1)b\,dt$$

Optical noises are derived from this.

$$P(n|n) = 1 - P((\neq n)|n) \approx 1 - P(n-1|n) - P(n+1|n) = 1 - (n(a+b)+c)dt$$

Master equation of photo(electro)n statistics

$$\frac{dP(n,t)}{dt} = -(n(a+b)+c)P(n,t) + ((n-1)a+c)P(n-1,t) + (n+1)bP(n+1,t)$$

Solution example for absorption only ($a = c = 0$): Poisson distribution

$$P(n) = e^{-\mu_0}\frac{\mu_0^n}{n!} \qquad \langle n \rangle \equiv \mu_0(t) = \mu_0(0)e^{-bt}$$

Moment generating function

$$M_n\left(e^{-s}\right)=\left\langle e^{-sn}\right\rangle=\sum_{n=-\infty}^{\infty}P(n)e^{-sn} \qquad M_x\left(e^{-s}\right)=\left\langle e^{-sx}\right\rangle=\int_{-\infty}^{\infty}p_x(x)e^{-sx}dx$$

Can be inverted by inverse Laplace or z transform ($e^{-s}=z^{-1}$)

MGF allows calculating all moments: $$\left\langle n^k\right\rangle,\left\langle x^k\right\rangle=(-1)^k\left.\frac{d^kM\left(e^{-s}\right)}{(ds)^k}\right|_{s=0}$$

Addition of statistically independent RVs: convolution of PDFs or multiplication of MGFs

MGF is now be applied to both sides of master equation. This results in ...

Solution of partial differential equation for MGF

$$\frac{\partial}{\partial t} M_n\left(e^{-s},t\right) = c\left(e^{-s}-1\right) M_n\left(e^{-s},t\right) - \left(a - be^{s}\right)\left(e^{-s}-1\right)\frac{\partial}{\partial s} M_n\left(e^{-s},t\right)$$

Derived from master equation of photo(electro)n statistics. Solution:

$$M_n\left(e^{-s},t\right) = \left(1+\mu\left(1-e^{-s}\right)\right)^{-N} M_n\left(1-\frac{G\left(1-e^{-s}\right)}{1+\mu\left(1-e^{-s}\right)},0\right)$$

MGF at (flight) time *t* is given in terms of the MGF at time 0 !

power gain during time *t*: $G = G(t) = e^{(a-b)t}$

number of modes: $N = c/a$

spontaneous emission factor: $n_{sp} = \frac{a}{a-b}$

mean noise photon number per mode: $\mu = n_{sp}(G-1)$

Discrete distributions (photoelectrons)

(type)	$P(n)\ (n \ge 0\ )$	$M_n\left(e^{-s}\right)$	$\langle n \rangle$	σ_n^2
Poisson (signal alone)	$e^{-\mu_0} \frac{\mu_0^n}{n!}$	$e^{-\mu_0\left(1-e^{-s}\right)}$	μ_0	μ_0
Central negative binomial (noise alone)	$\binom{n+N-1}{n} \frac{\mu^n}{(1+\mu)^{n+N}}$	$\frac{1}{\left(1+\mu\left(1-e^{-s}\right)\right)^N}$	$N\mu$	$N\mu(\mu+1)$
Noncentral negative binomial, Laguerre (signal + noise) General case	$\frac{\mu^n e^{-\frac{\mu_0}{1+\mu}}}{(1+\mu)^{n+N}} L_n^{N-1}\left(\frac{-\mu_0}{\mu(1+\mu)}\right)$	$\frac{e^{\frac{-\mu_0\left(1-e^{-s}\right)}{1+\mu\left(1-e^{-s}\right)}}}{\left(1+\mu\left(1-e^{-s}\right)\right)^N}$	$\mu_0 + N\mu$	$N\mu(\mu+1) + (2\mu+1)\mu_0$

Poisson transformation and normalization

Assume that the probability distribution of the photon number *n* can be expressed by the **Poisson transform** of the PDF of a continuous nonnegative RV *x*:

$$P(n)=\int_0^\infty p_x(x)e^{-x}\frac{x^n}{n!}dx$$

For $G\to\infty$ no limit of $P(n)$ is found because the mean photon number scales with G.

A normalized variable $\tilde{x}=x/G$ has a $p_{\tilde{x}}(\tilde{x})$ which depends only weakly on G and allows finding $\lim_{G\to\infty} p_{\tilde{x}}(\tilde{x})$.

$$p_x(x)dx=p_{\tilde{x}}(\tilde{x})d\tilde{x} \qquad p_{\tilde{x}}(\tilde{x})=Gp_x(\tilde{x}G)$$

$$P(n)=\int_0^\infty p_{\tilde{x}}(\tilde{x})e^{-\tilde{x}G}\frac{(\tilde{x}G)^n}{n!}d\tilde{x}$$

This is the continuous form of $P(n)=\sum P(n|(\tilde{x}_iG))P(\tilde{x}G=\tilde{x}_iG)$ where the conditional probability $P(n|(\tilde{x}_iG))$ is that of a Poisson distribution with a mean $\tilde{x}_iG$. We find:

$$\lim_{G\to\infty} M_n\left(e^{-s/G}\right)=\lim_{G\to\infty}\sum_{n=0}^{\infty}e^{-(s/G)n}P(n)=\lim_{G\to\infty}\sum_{n=0}^{\infty}e^{-(s/G)n}\int_0^\infty p_{\tilde{x}}(\tilde{x})e^{-\tilde{x}G}\frac{(\tilde{x}G)^n}{n!}d\tilde{x}$$

$$=\int_0^\infty p_{\tilde{x}}(\tilde{x})\lim_{G\to\infty}e^{-\tilde{x}G}\sum_{n=0}^{\infty}\frac{\left(e^{-s/G}\tilde{x}G\right)^n}{n!}d\tilde{x}=\int_0^\infty p_{\tilde{x}}(\tilde{x})\lim_{G\to\infty}e^{-\tilde{x}G}e^{e^{-s/G}\tilde{x}G}d\tilde{x}$$

$$=\int_0^\infty p_{\tilde{x}}(\tilde{x})\lim_{G\to\infty}e^{-\tilde{x}G\left(1-e^{-s/G}\right)}d\tilde{x}=\int_0^\infty p_{\tilde{x}}(\tilde{x})\lim_{G\to\infty}e^{-\tilde{x}G(s/G)}d\tilde{x}=\int_0^\infty p_{\tilde{x}}(\tilde{x})e^{-s\tilde{x}}d\tilde{x}=M_{\tilde{x}}\left(e^{-s}\right)$$

$p_{\tilde{x}}(\tilde{x})$ is obtained by backtransforming $M_{\tilde{x}}\left(e^{-s}\right)$.

Continuous distributions (intensity, power, photocurrent)

(type)	$p_{\tilde{x}}(\tilde{x})\ (\tilde{x} \geq 0)$	$M_{\tilde{x}}(e^{-s})$	$\langle \tilde{x} \rangle$	$\sigma_{\tilde{x}}^2$
Constant (signal alone)	$\delta(\tilde{x} - \tilde{\mu}_0)$	$e^{-\tilde{\mu}_0 s}$	$\tilde{\mu}_0$	0
Central χ_{2N}^2, Gamma (noise alone)	$\frac{1}{\Gamma(N)} \tilde{\mu}^N \tilde{x}^{N-1} e^{-\tilde{x}/\tilde{\mu}}$	$\frac{1}{(1+\tilde{\mu}s)^N}$	$N\tilde{\mu}$	$N\tilde{\mu}^2$
Noncentral χ_{2N}^2 (signal + noise) General case	$\frac{\tilde{x}^{(N-1)/2} e^{-(\tilde{\mu}_0+\tilde{x})/\tilde{\mu}}}{\tilde{\mu}_0^{(N-1)/2} \tilde{\mu}} \cdot I_{N-1}\left(2\sqrt{\tilde{x}\tilde{\mu}_0}/\tilde{\mu}\right)$	$\frac{e^{\frac{-\tilde{\mu}_0 s}{1+\tilde{\mu}s}}}{(1+\tilde{\mu}s)^N}$	$\tilde{\mu}_0 + N\tilde{\mu}$	$N\tilde{\mu}^2 + 2\tilde{\mu}\tilde{\mu}_0$

Eliminating and adding shot noise

$P(n)$: Poisson distribution, Central negative binomial distribution, Noncentral negative binomial distribution	$\Rightarrow M_n\left(e^{-s}\right) \Rightarrow$ $M_{\tilde{x}}\left(e^{-s}\right) = \lim_{G\to\infty} M_n\left(e^{-s/G}\right) \Rightarrow$ Eliminate shot noise by amplification, normalize with respect to G.	$p_{\tilde{x}}(\tilde{x})$: Constant (Dirac function), Central χ^2 distribution, Noncentral χ^2 distribution
	$\Leftarrow P(n) = \int_0^\infty p_{\tilde{x}}(\tilde{x}) e^{-\tilde{x}G} \frac{(\tilde{x}G)^n}{n!} d\tilde{x} \Leftarrow$ Add shot noise, undo normalization ($x = \tilde{x}G$).	

Direct detection receiver model (1)

Assume independent zero-mean Gaussian noise variables with equal variances!

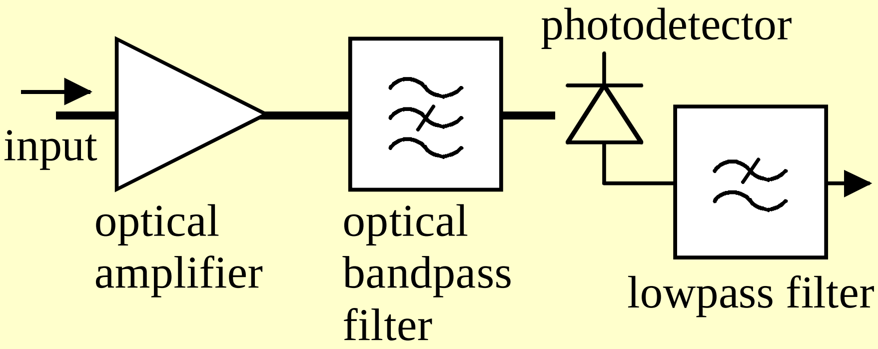
Bandpass filter has rectangular impulse response of duration τ_1. Electrical field at its output:

$$\underline{\tilde{\mathbf{E}}}(t)=\left(\left(\sqrt{\tilde{\mu}_0/M}+\tilde{u}_1+j\tilde{u}_2\right)\underline{\mathbf{e}}_1+\left(\tilde{u}_3+j\tilde{u}_4\right)\underline{\mathbf{e}}_2\right)e^{j\omega t}$$

Photocurrent: $$\left|\underline{\tilde{\mathbf{E}}}(t)\right|^2=\left(\sqrt{\tilde{\mu}_0/M}+\tilde{u}_1\right)^2+\tilde{u}_2^2+\tilde{u}_3^2+\tilde{u}_4^2$$

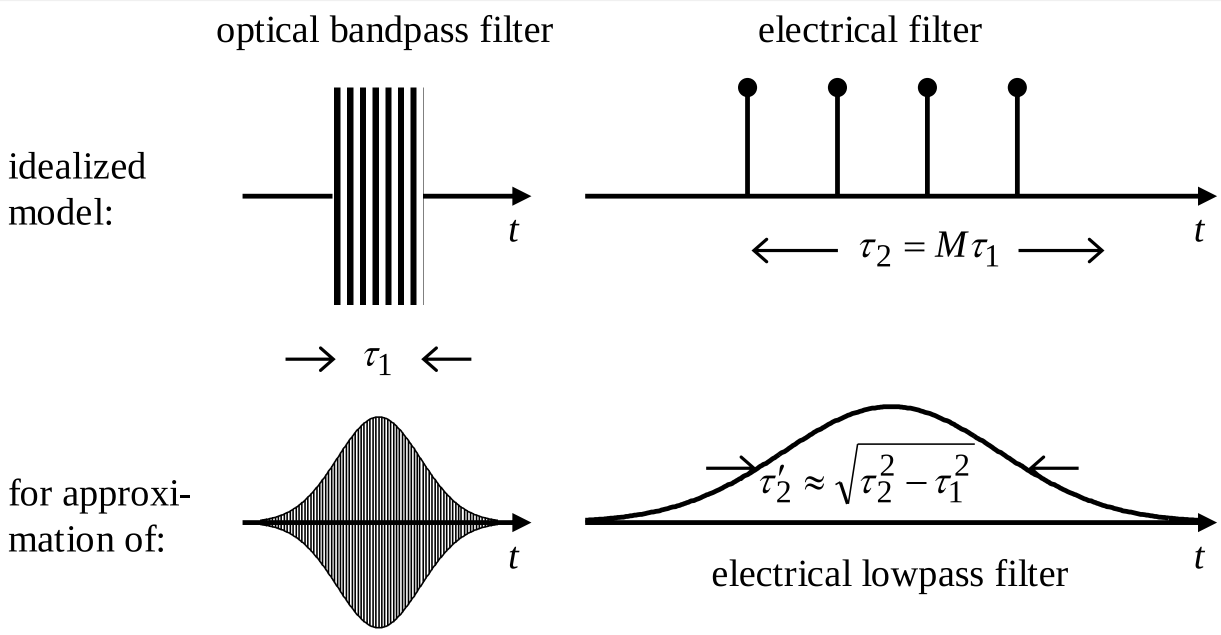
A lowpass filter with a continuous impulse response of length τ_2 or better $\sqrt{\tau_2^2-\tau_1^2}$ is modeled as a (ficticious, infinite bandwidth) lattice filter having M Dirac impulses spaced by τ_1 each. The signal at its output is χ^2 distributed with $4M = 2N$ degrees of freedom:

$$\tilde{x}=\sum_{i=1}^{M}\left|\underline{\tilde{\mathbf{E}}}(t+i\tau_1)\right|^2$$

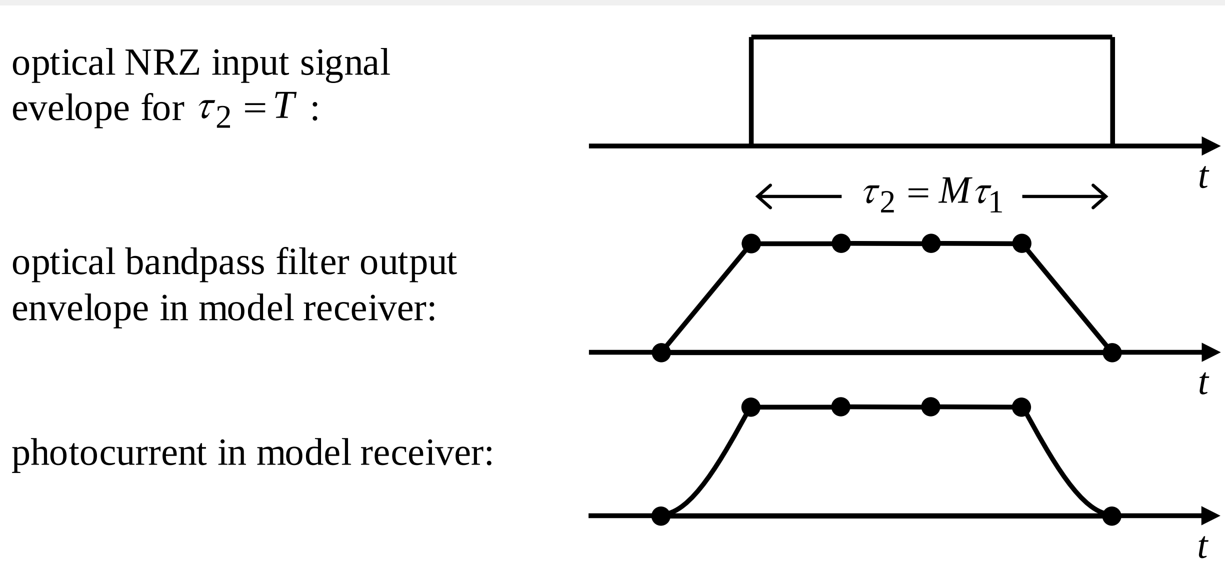


N modes = p polarizations · M samples

Direct detection receiver model (2)



Optical and electrical impulse responses in optical receiver



Optical field envelopes and photocurrent for idealized model sketched above

Samples with statistically independent noises are marked by •.

Understanding optical amplifier and photodetection

- Signal and noise together have a noncentral negative binomial photoelectron distribution. It contains optical amplifier noise + shot noise.
- For $G \to \propto$, shot noise of detection becomes negligible. We get a noncentral χ^2 intensity distribution. Its noise is only optical amplifier noise.
- Noncentral χ^2 intensity distribution occurs in DD RX with Gaussian field noise.
- The only possible explanation is:

  Amplifier adds Gaussian field noise in phase and in quadrature (wave aspect).
  Detection adds shot noise (particle aspect).

  Duality: Particles contain no wave. Wave contains no particles. Connection: $W = hf$
  This avoids contradictions (interference, sub-photon energies, ...)!
- Gaussian field noise + shot noise characterize optical noise and NF completely.
- The known noise distribution allows calculating bit error ratio exactly. This works better than the Gaussian approximation for which F_{pnf} was derived.
- Field noise occurs $\Rightarrow$ field amplitudes must be considered, not „power amplitudes".

$\Rightarrow$ We need linear receivers! Coherent optical receivers are linear in amplitudes!

Optical field noise as amplified zero point fluctuations

Amplifier adds Gaussian field noise in phase and in quadrature (wave aspect).
Detection adds shot noise (particle aspect).

The above was derived from: $P(n|n-1) = ((n-1)a + c)dt \qquad P(n|n+1) = (n+1)bdt$

Resulting field noise may also be understood as amplified zero-point energy $hf/2$:
Zero-point energy works like 1 / 0 detectable photons for emission / absorption.

$$P(n+1|n) = (n+1 \cdot N)a\,dt \qquad P(n-1|n) = (n+0)bdt \qquad N = c/a$$ noisy modes

Emission and absorption combined: $d\langle n \rangle / dt = (\langle n \rangle + N)a - \langle n \rangle b \quad \Rightarrow$

$$\langle n(t) \rangle = G\langle n(0) \rangle + N n_{sp}(G-1) \qquad G = e^{(a-b)t} \qquad n_{sp} = \frac{a}{a-b}$$

$P_{out} = GP_{in} + N n_{sp}(G-1)hfB_o$ This is equivalent to our earlier results. Only probability distribution of field noise and existence of shot noise are not derived here.

If electron occupation probabilities in upper and lower states are f_2, f_1:

$$a = f_2(1-f_1) \qquad b = f_1(1-f_2) \qquad G = e^{(f_2-f_1)t} \qquad n_{sp} = \frac{f_2(1-f_1)}{f_2-f_1}$$

See also: M. Yamada, „Theory of of Semiconductor Lasers“, Springer, 2014