

# Frequency and Phase Estimation for Coherent QPSK Transmission With Unlocked DFB Lasers

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**Abstract**—This letter presents a hardware-efficient frequency estimator and an advanced phase estimation algorithm capable of tracking the phase noise of a 10-GBaud optical quadrature phase-shift-keying transmission system with standard distributed-feedback lasers in the presence of a frequency mismatch up to 1.2 GHz. This algorithm allows us to implement a digital coherent receiver without an analog frequency control circuit.

**Index Terms**—Frequency estimation (FE), optical communication, phase estimation (PE), quadrature phase-shift keying (QPSK), synchronous detection.

## I. INTRODUCTION

COHERENT detection combined with complex modulation formats like quadrature phase-shift keying (QPSK) has been the subject of several recent real-time transmission experiments [1]–[4] directed to increase the spectral efficiency and tolerance to chromatic and polarization-mode dispersion. Common optical phase-locked loop (OPLL) concepts [5] cannot ensure proper functionality due to the linewidth of the preferred distributed-feedback (DFB) lasers and are replaced by digital feed-forward estimators [6]. Such a phase estimation (PE) algorithm requires that the intermediate frequency (IF) between the transmit and the local oscillator (LO) laser is small compared to the data rate. Previously, it was shown that a digital frequency estimator (FE) and compensator preceding the essential receiver (phase and data recovery) allows us to compensate for frequency mismatch up to the gigahertz range for a symbol rate of 2.2 GBaud [2]. To the best of our knowledge, such a preprocessing unit is the only published strategy to cope with potentially high IFs that occur if there is no automatic LO frequency control.

In this letter, we present a hardware-efficient FE approach. Furthermore, we investigate the influence of large IFs on the PE

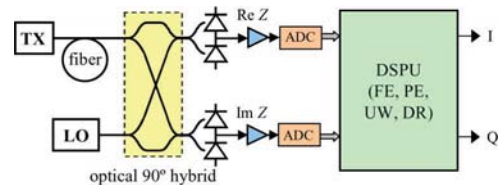


Fig. 1. Coherent QPSK receiver with digital signal processing unit (DSPU). The DSPU contains FE, PE, and phase unwrapping (UW) for an angle-based data recovery (DR).

algorithm called nonlinear compensation function (NCF) [7]. Improvements of the algorithm that use an estimated intermediate angular frequency (EIAF) allow simultaneous compensation of frequency mismatch and phase noise, and an FE-based preprocessing can be avoided. In a polarization-multiplexed receiver with integrated correlation-based polarization control unit [3], [8], the presented frequency and phase estimators can also be employed advantageously.

Fig. 1 shows the coherent receiver with DSPU. FE, PE, and phase unwrapping are auxiliary functions for the data recovery.

## II. FREQUENCY ESTIMATION

The sampled received symbols of a coherent QPSK transmission system can be described as

$$Z(k) = c(k)e^{j(\omega_{IF}kT + \varphi(k))} + n(k) \quad (1)$$

where  $Z(k)$  is a complex number consisting of the sent QPSK symbol  $c(k)$  multiplied by a time-variant phasor, and additional noise  $n(k)$ . The phasor can be separated into an IF part  $e^{j\omega_{IF}kT}$  and a random part  $e^{j\varphi(k)}$  representing the phase noise. If the phase noise is dominating and the IF is small, a separate FE is not necessary because the phase estimator alone can track the unseparated phasor. Otherwise, using the EIAF for preprocessing according to [2] would yield

$$Z'(k) = Z(k)e^{-j\hat{\omega}_{IF}kT} \approx c(k)e^{j\varphi(k)} + n'(k) \quad (2)$$

and the subsequent PE would receive an input signal modulated with a residual IF close to zero. The additional channel noise  $n(k)$  is also rotated to  $n'(k)$  but does not change its magnitude.

For FE, a variety of algorithms is available [9]. For example, the algorithm of Rife and Boorstyn [10] is based on finding the maximum in a discrete spectrum.

Leven *et al.* [11] proposes complex multiplication of the received symbol with its complex conjugate predecessor. The effect of the QPSK modulation is removed by raising this product to its fourth power. Li *et al.* [12] employ a predecision-based

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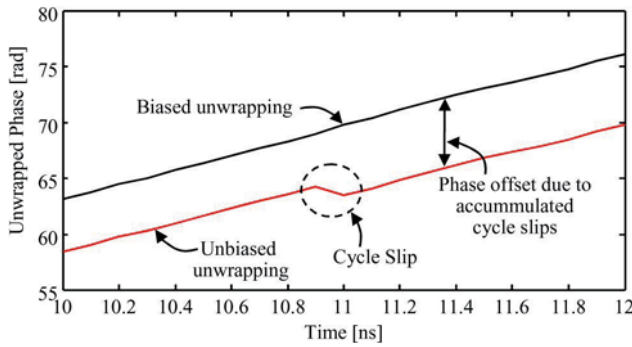


Fig. 2. Unwrapping the estimated phase with and without bias.

error detector in their angle-based approach. The method presented in this letter avoids both complex calculation and predecision.

The proposed estimator can find  $\hat{\omega}(k)$  simply from the arguments  $\psi(k)$  of the received symbol sequence  $Z(k)$ . The instantaneous intermediate angular frequency times the sampling period  $T$  is the phase increment on the shortest possible physical path

$$\omega_{\text{IF}}(k)T \approx \delta(k) := \left( \left( \psi(k) - \psi(k-1) + \frac{\pi}{4} \right) \bmod \frac{\pi}{2} \right) - \frac{\pi}{4}. \quad (3)$$

Like the spectrum in [10], the histogram of a sufficiently large set of  $\delta(k)$  values contains a maximum representing the IF. The instantaneous values  $\omega_{\text{IF}}(k)$  may be averaged to obtain an estimate, which works well for small IF values. For a higher IF, a recursive FE should be employed (averaging is denoted by  $\langle \cdot \rangle$ ):

$$\hat{\omega}_{\text{IF,new}} := \hat{\omega}_{\text{IF,old}} + h \left\langle \frac{\delta(k)}{T} - \hat{\omega}_{\text{IF,old}} \right\rangle. \quad (4)$$

Starting with  $\hat{\omega}_{\text{IF,old}} = 0$ , the recursive FE is able to reach IF values up to  $|\omega_{\text{IF}}| = \pi/4T$ . The parameter  $h \in [0, 1]$  can be used to slow down the estimation. The update rate for the FE can be chosen much slower than the DSPU system clock. In order to reduce the influence of noise and distortions,  $\psi$  in (4) might be replaced by estimated phase values  $\hat{\phi}$  for an *a posteriori* estimation.

### III. PHASE ESTIMATION AND UNWRAPPING

A Viterbi phase estimator [13] uses a modulation-free signal

$$Y(k) = |Z(k)|^p e^{j4\psi(k)}, \quad p \in \{0, 2, 4\}. \quad (5)$$

Choosing  $p = 0$  yields a normalized input signal  $Y(k) = e^{j4\psi(k)}$  which is easy to calculate and contains reduced noise and distortions. Similar to the predecision-based phase-estimator proposed by Tao *et al.* [14], the use of complex multipliers for modulation removal is avoided. The continuous PE uses  $2N + 1$  values of the complex signal  $Y(k)$  to generate a moving average. Unlike a block phase estimator [2], [4], [10], the continuous PE approach [1], [3], [6]–[9] delivers separate estimated phase values for each received symbol. Choosing a high value of  $N$  results in a smooth estimated phase but the low bandwidth of this filter limits its tracking capability. On

the other hand, small values of  $N$  lead to higher sensitivity of the phase estimator against noise. Weighted averaging

$$\langle Y(k) \rangle = \frac{1}{2N + 1} \sum_{n=k-N}^{k+N} g_{n-k} Y(n) \quad (6)$$

with symmetrically decaying coefficients  $g_0 \geq g_{\pm 1} \geq g_{\pm 2} \geq \dots \geq g_{\pm N} > 0$  outperforms simple unweighted averaging. The combination of (5) and (6) was called NCF in [7]. The results in [7] were obtained with rounded 3-bit integer weights:  $g_0 = 8$ ,  $g_{\pm 1} = 4$ ,  $g_{\pm 2} = 3$ ,  $g_{\pm 3} = 2$ ,  $g_{\pm 4} = g_{\pm 5} = 1$ .

In the presence of IF estimation and the absence of pre-processing, it is possible to modify the NCF with a frequency compensation (FC) resulting in a summation with complex weighting coefficients

$$\langle Y(k) \rangle = \frac{1}{2N + 1} \sum_{n=k-N}^{k+N} g_{n-k} e^{j\hat{\omega}_{\text{IF}}(k-n)T} Y(n). \quad (7)$$

The weighted contributions are aligned to each other in the complex plane. Finally, the averaged signal from (6) or (7) has to be converted into an estimated phase where the fourfold ambiguity of  $\sqrt[4]{\langle Y(k) \rangle}$  has to be resolved, e.g., by

$$\hat{\phi}(k) := \frac{1}{4} \arg \langle Y(k) \rangle \bmod \frac{\pi}{2}. \quad (8)$$

Such an estimated phase that is always chosen from  $[0, \pi/2]$  or another arbitrary  $\pi/2$ -wide fixed interval is called a wrapped phase. The wrapped estimated phase has either to be unwrapped [16] or jump numbers have to be generated for the data recovery [6]. In both cases, a high IF value could lead to cycle slips, i.e., tracking of the physical phase would fail. A remedy against this problem is to use the estimated frequency as a direction bias.

Fig. 2 shows unbiased and biased unwrapping (BU) of the estimated phase for a 10-GBaud transmission system with an IF of 1 GHz. Only the BU yields a smooth curve with a slope according to the direction bias given by the EIAF. The wrapped phase typically contains nonphysical phase jumps that lead to demodulation errors. But it is possible to detect these jumps and encode them into a 2-bit quadrant jump number and take the estimated IF bias into account

$$n_j(k) = \begin{cases} 0, & |\hat{\phi}(k) - \hat{\phi}(k-1) - \hat{\omega}_{\text{IF}}T| \leq \frac{\pi}{4} \\ 1, & \hat{\phi}(k) - \hat{\phi}(k-1) - \hat{\omega}_{\text{IF}}T < -\frac{\pi}{4} \\ -1, & \hat{\phi}(k) - \hat{\phi}(k-1) - \hat{\omega}_{\text{IF}}T > \frac{\pi}{4} \end{cases}. \quad (9)$$

Without the bias term  $\hat{\omega}_{\text{IF}}T$ , the quadrant jump numbers are equivalent to definitions from [6] and [14].

### IV. SIMULATION RESULTS

The presented PE algorithms NCF, NCF with FC, and NCF with FC and BU were compared to a simple differential (also called asynchronous) receiver in a Monte Carlo simulation. Combination of the differential receiver with FE-based preprocessing was also simulated. Bit-error-rate (BER)/signal-to-noise ratio (SNR) curves with  $10^6$  symbols for each BER value were generated for a 10-Gbaud transmission system with phase noise equivalent to a linewidth to symbol rate ratio of 0.001 and the IF varied from zero up to 1.24 GHz. This is close to the 1.25-GHz limit beyond which QPSK

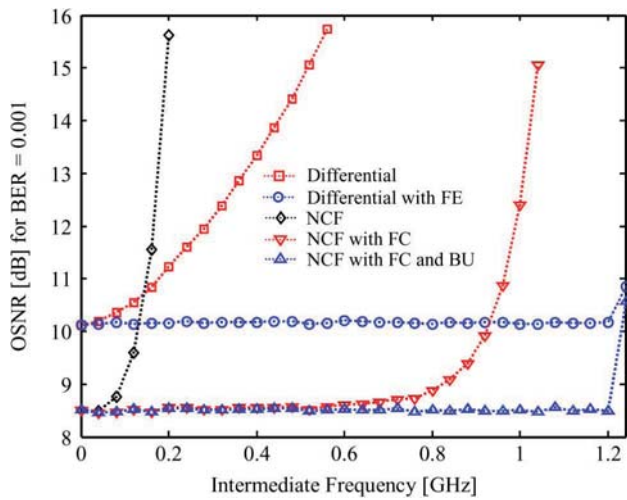


Fig. 3. Simulated SNR curves.

quadrants would be falsified. Higher IF values would require a data aided approach because any nondata-aided FE would converge towards a wrong downconverted IF value, similar to the well-known alias effect in analog to digital conversion.

Fig. 3 shows the required SNR values for a BER smaller than  $10^{-3}$ . If the IF is smaller than 100 MHz, the NCF PE alone is sufficient for about 1.5-dB SNR gain compared to differential receiver. For higher frequencies, the improvements allow to maintain the PE quality up to 800 MHz (FC) or even 1.2 GHz (FC and BU). This improved IF tolerance is considered to be sufficient for a transmission system with unlocked DFB lasers.

## V. SUMMARY

We have presented a very powerful and hardware-efficient FE algorithm versatile for both the usual preprocessing and also for an improved PE algorithm. For best performance, the estimated frequency should also be used as a bias in a generalized unwrapping algorithm. Simulations indicate that the improved algorithms allow to run a 10-GBaud QPSK transmission system with DFB lasers that have a frequency mismatch of up to 1.2 GHz.

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