Exercise 9: Three-Phase Line Commutated Rectifier

A DC-motor is supplied by a three-phase line commutated rectifier. Its topology is shown in figure 1. This rectifier is controlled by the most common method by turning on the thyristors at a predetermined phase of the sinusoidal voltage waveform. The average value and the shape of the output voltage v_D depends on the firing angle α.

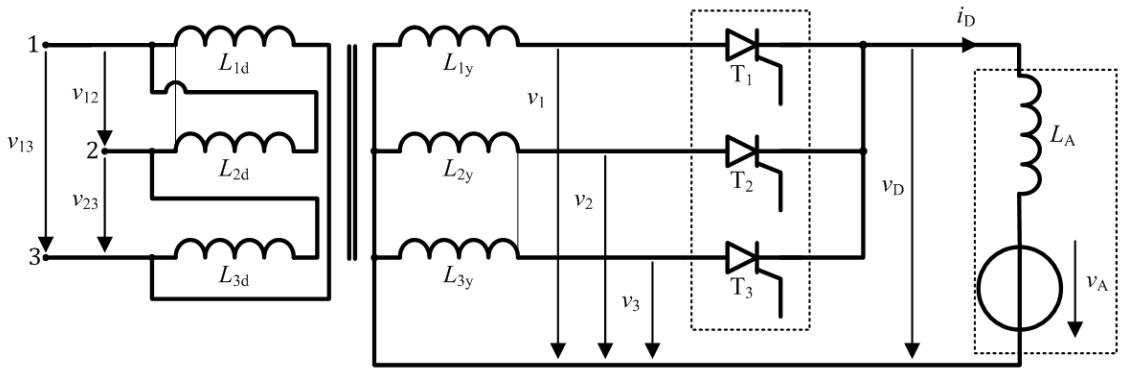


Figure 1: Three-phase half-wave controlled rectifier

Note the following hints:

- Please use the appended waveform graphs
- The following addition theorems and coherencies will be needed in this exercise:

$$\sin(x+y) - \sin(x-y) = 2\cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

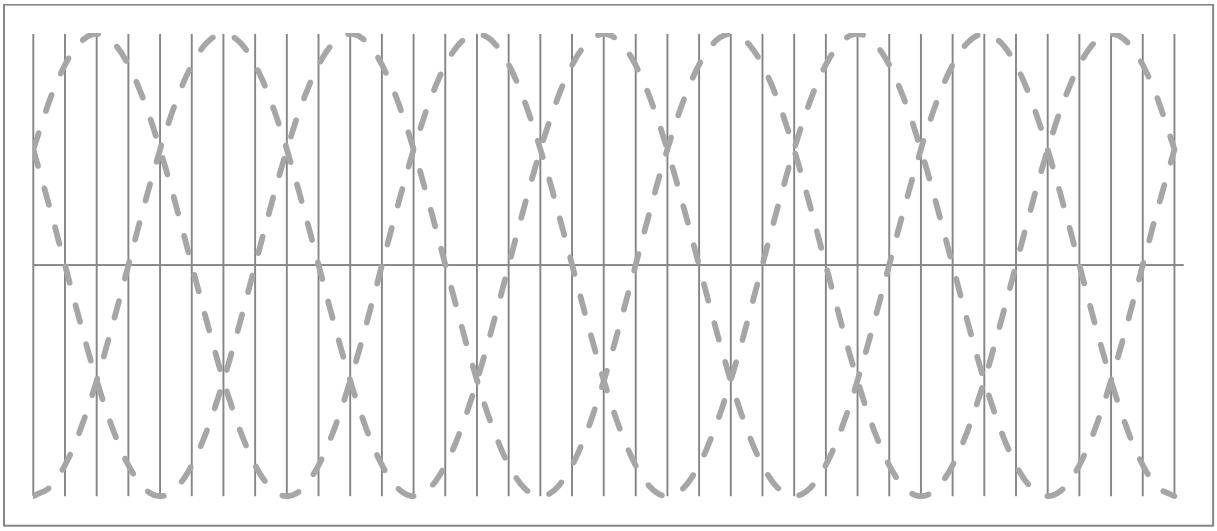
$$\sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

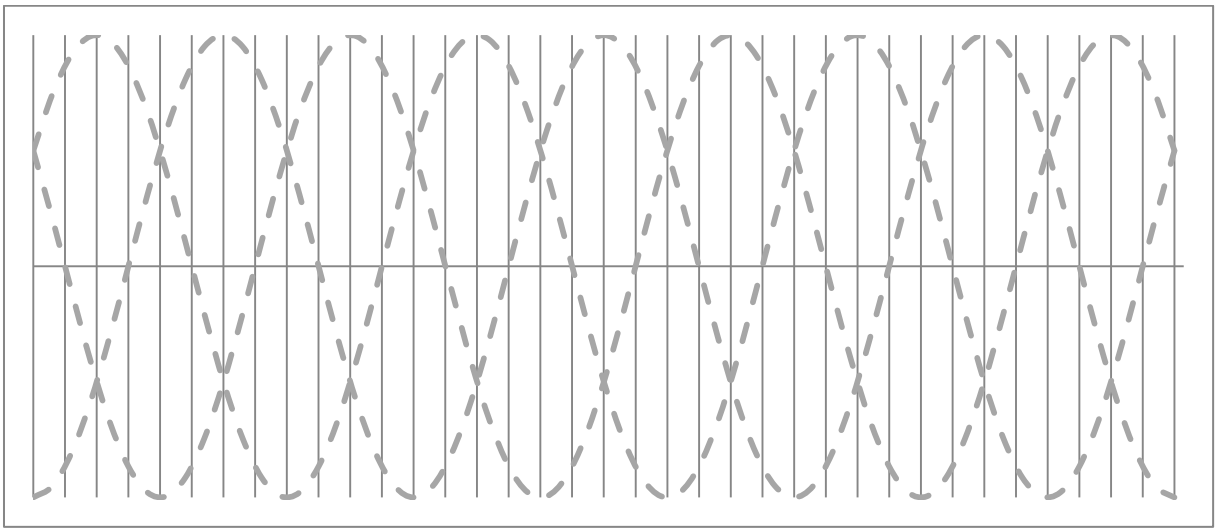
$$\cos\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

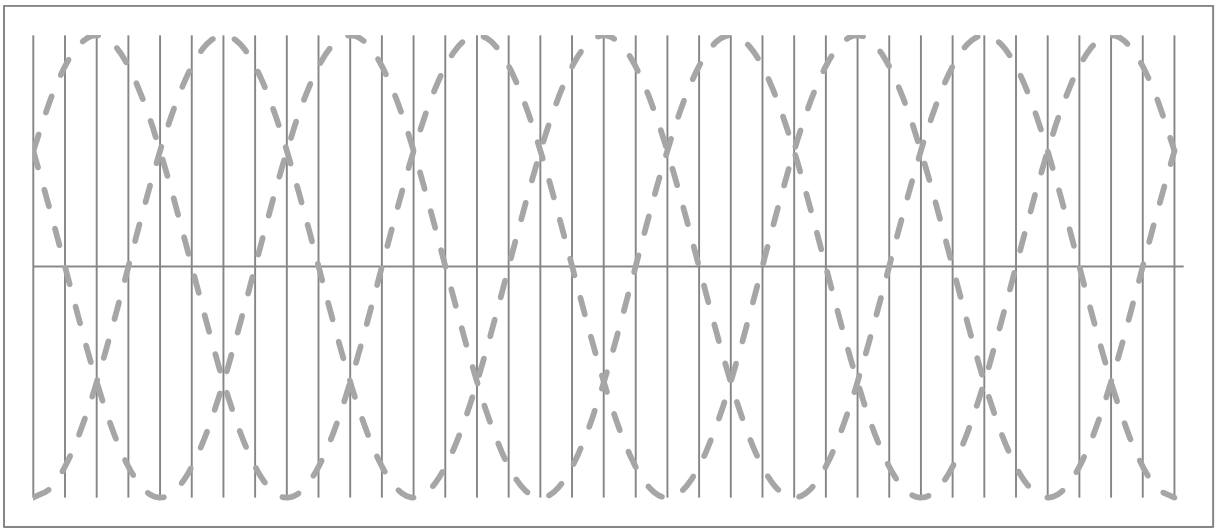
1. Draw the waveform of the output voltage $v_\mathrm{D}(t)$ for $\alpha = 0$ degrees, $\alpha = 90$ degrees and $\alpha = 180$ degrees. Assume that the inductor current i_D is ideally smooth.
2. Set up a general formula for the output current $i_\mathrm{D}(t)$.
3. What are the steady state conditions for $i_\mathrm{D}(t)$?
4. Draw the output current waveform $i_\mathrm{D}(t)$ over a couple of pulse periods for $\alpha = 0$ degrees, $\alpha = 90$ degrees and $\alpha = 180$ degrees. Calculate the phase cutting angle α for the case of an undistorted output current flow. Assume that the steady state conditions for $i_\mathrm{D}(t)$ are fulfilled.
5. Draw the waveform of the output voltage $v_\mathrm{D}(t)$ and the voltage waveform across one of the thyristors for $\alpha = 30$ degrees.

6. Determine the extreme values of $i_{\mathrm{D}}(t)$ when the phase cutting angle α is treated as an independent variable. For which angle α does the output current start with a horizontal tangent?
7. Calculate the phase cutting angle α_{crit} for the case of maximum output current ripple.
8. Calculate the amplitude of the maximum current ripple.
9. Determine the average value of the output current $I_{\mathrm{d_{min}}}$ at the given phase angle $\alpha = \alpha_{\mathrm{crit}}$ and the case that the rectifier is operating in boundary conduction mode. Calculate the value of the relative current ripple $\Delta i_{\mathrm{D}}/I_{\mathrm{d_{min}}}$.
10. Use the results from problem 9 and derive a formula to calculate the inductance L_{A}. Calculate the value of L_{A} for a given $I_{\mathrm{d_{min}}} = 1\ \mathrm{A}$ and a given RMS voltage value of $V_{1,2,3} = 230\ \mathrm{V}$ at the transformer secondary side.

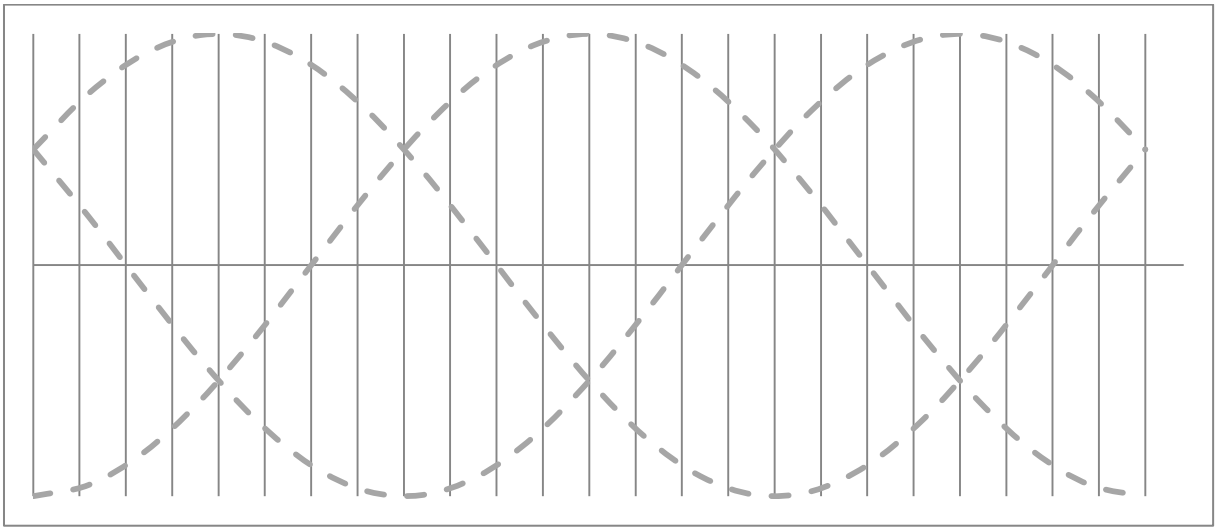
Auxiliary sheet for problem 1

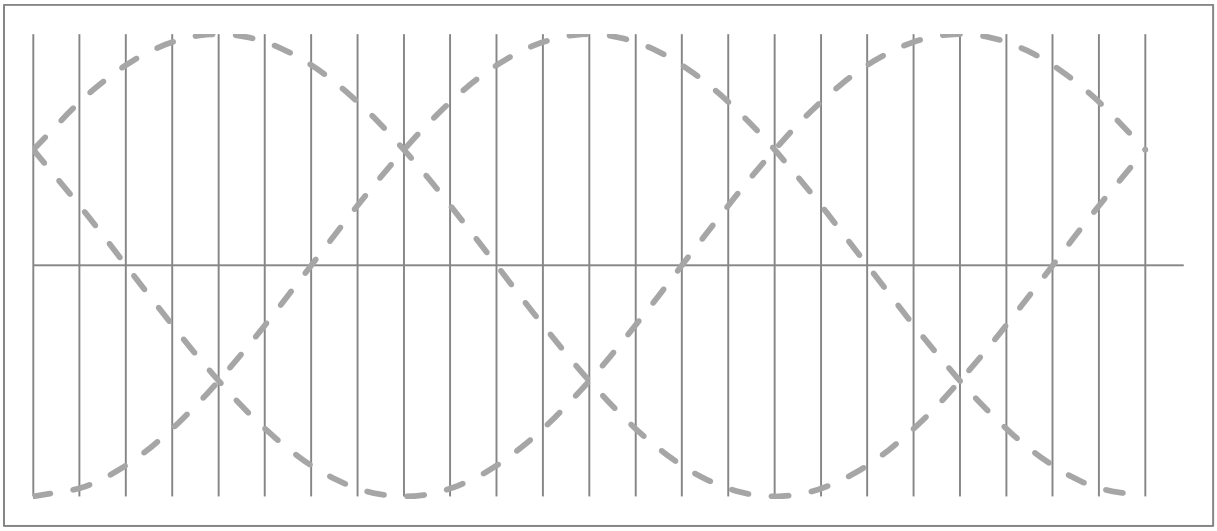




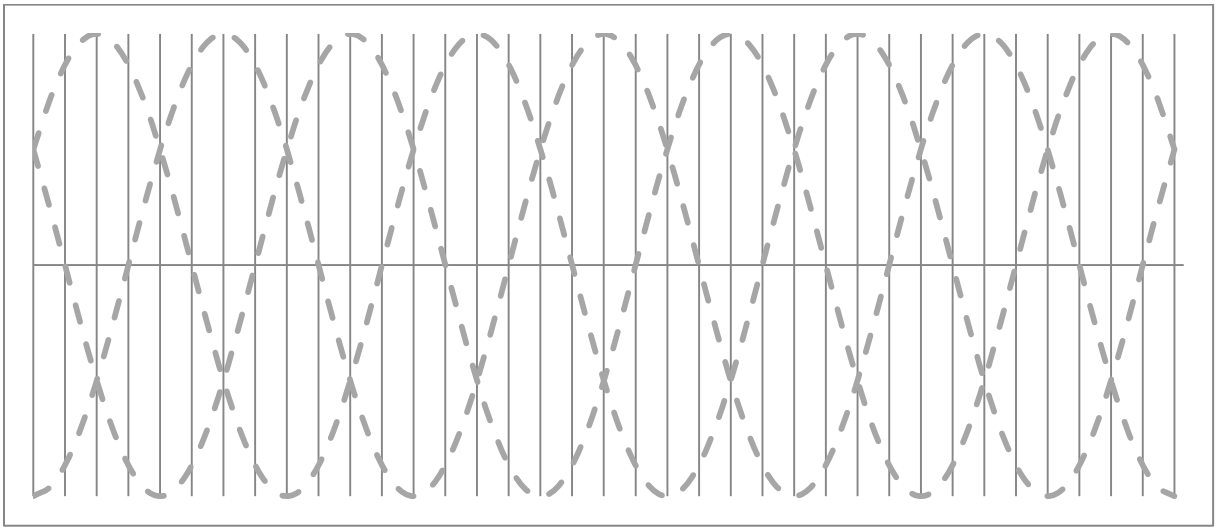


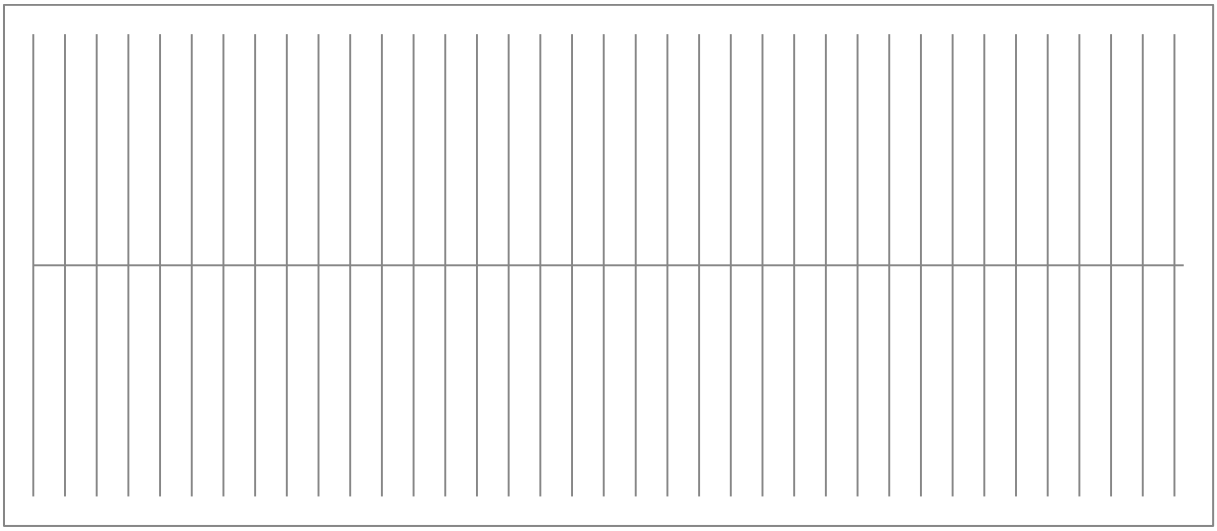
Auxiliary sheet for problems 3 and 6



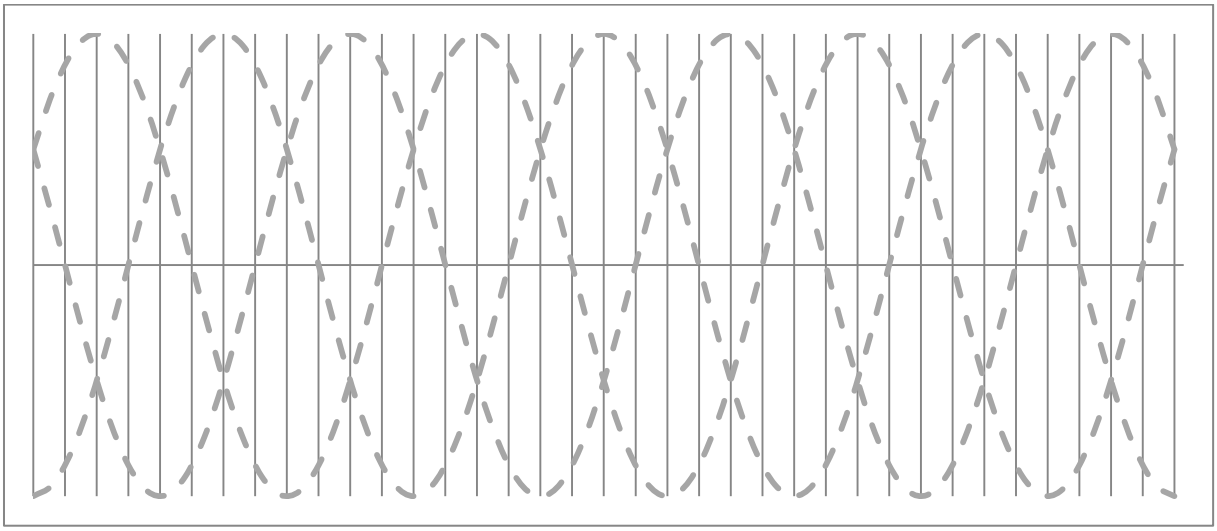


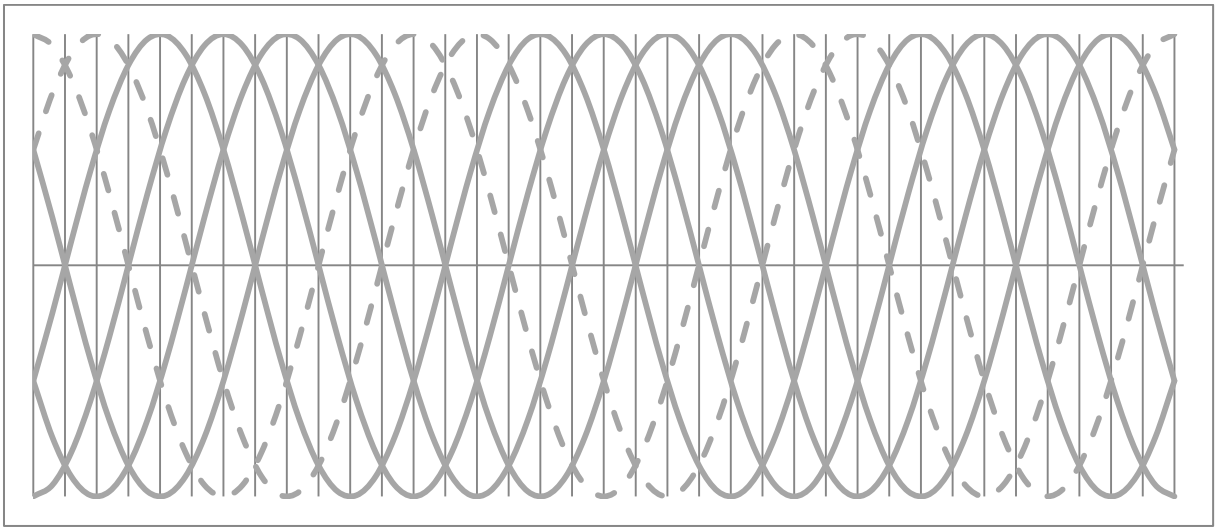
Auxiliary sheet for problem 4





Auxiliary sheet for problem 5





Auxiliary sheet for problem 8