

Power Electronics

13.09.2016

Last Name:						Student Number:					
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First Name:											
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Study Program:						<input type="checkbox"/> Professional Examination <input type="checkbox"/> Performance Proof					
Task: (Credits)	1 (20)	2 (20)	3 (20)	4 (20)		Total (80)					Mark

Duration: 120 minutes

Permitted auxiliaries:

- Nonprogrammable calculator without graphic display
- Drawing material (circle, triangle, ruler, pens...)

Please note the following remarks:

- You may only attend the exam if you have registered for it in the system PAUL. If you participate without registration your generated results will not be evaluated and accepted.
- Please hold your student identity card with photograph in readiness!
- Please label every blank sheet of paper with your name and your student number. For every task please use a new blank sheet of paper. Please do not use pencils and red pens.
- Every numerical calculation has to be furnished with units. Non-compliance will lead to point deduction.
- All approaches have to be documented in a comprehensible way! Specifying a single value as final result without a recognizable approach will not be evaluated.

Good Luck!

Task 1: Buck Converter**(20 Credits)**

For an automotive application a buck converter is used. The output capacitor C is very large so that the output voltage ripple can be neglected. All components are assumed to be ideal. Please assume steady state for the complete task.

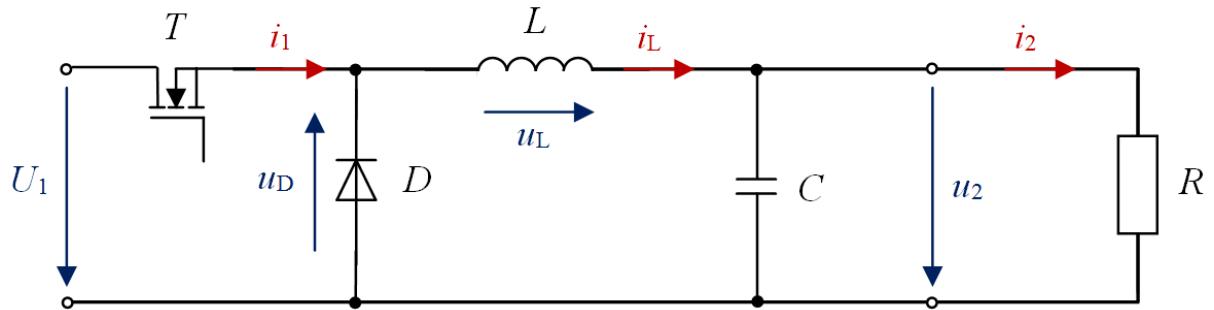


Figure 1.1: Circuit diagram of buck converter with resistive load

The converter operates in discontinuous conduction mode (DCM). The chosen and adjusted values are given as follows:

- Input voltage: $U_1 = 12 \text{ V}$
- Load resistor: $R = 10 \Omega$
- Duty cycle: $D_{\text{DCM}} = 0.2$
- Switching frequency $f_S = 50 \text{ kHz}$

Furthermore the plot of the input current i_1 over two switching periods is shown in Figure 1.2:

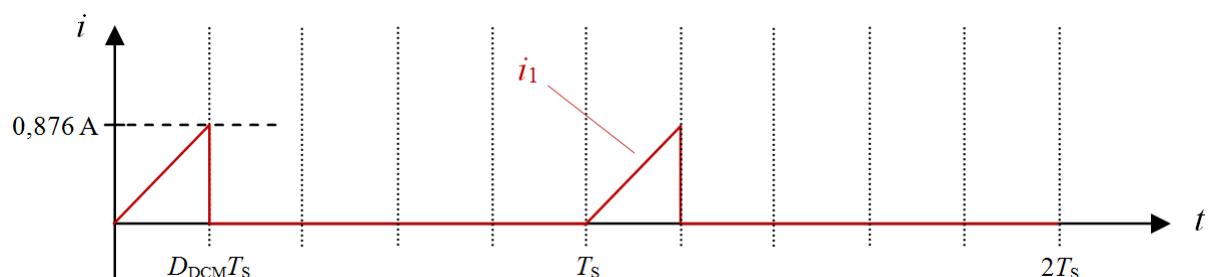
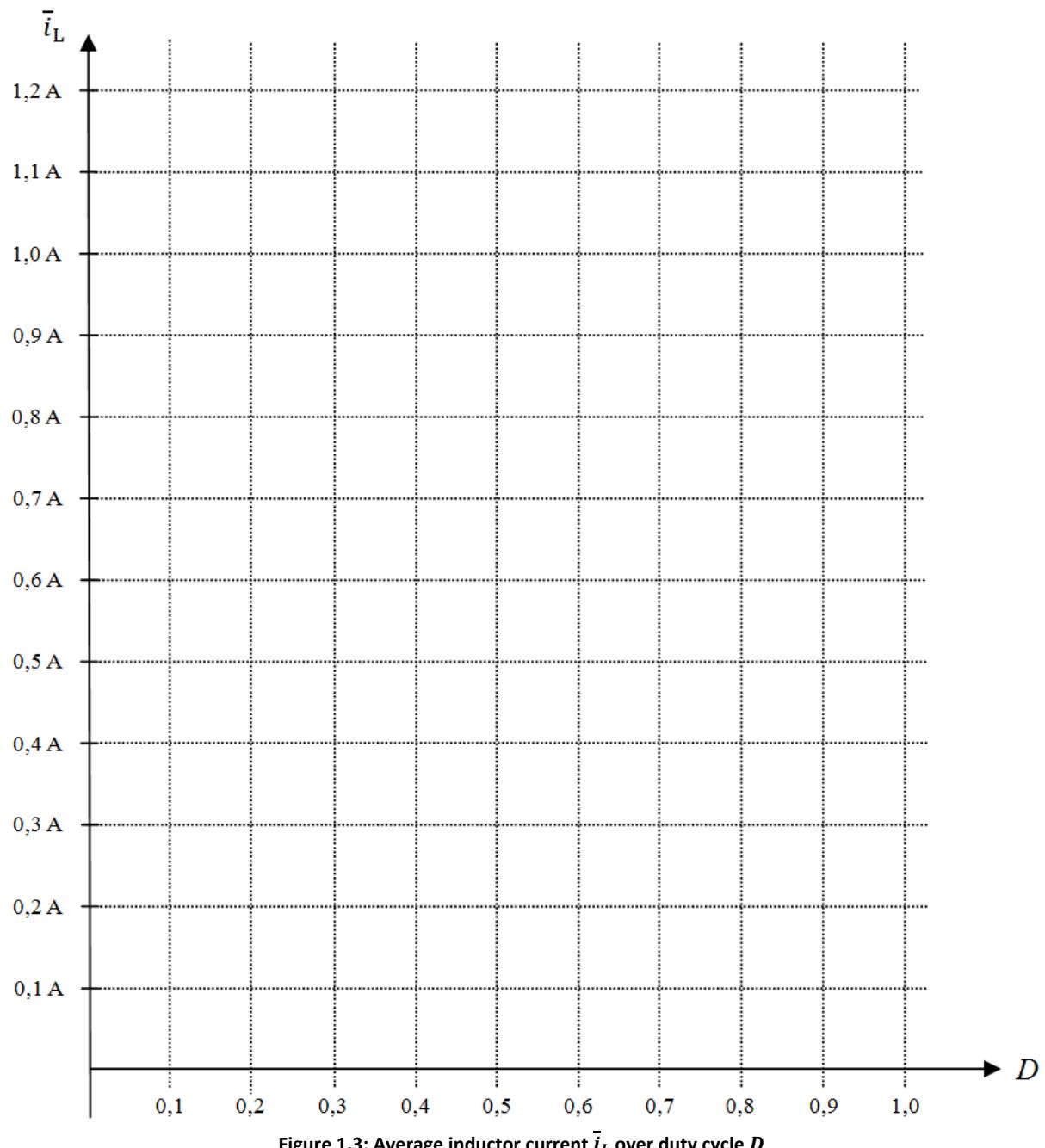


Figure 1.2: Input current i_1 over two switching periods

- 1.1** Calculate the average value \bar{i}_1 of the input current i_1 . Determine the power $P_1 = P_2 = P$ which has to be provided by the DC voltage source.
- 1.2** Calculate the values of the output voltage $u_2 = U_2$ and the output current $i_2 = I_2$.
- 1.3** Determine the value of the inductance L .
- 1.4** Determine the current conducting duration T_{off} of the diode. Draw the inductor current i_L in the diagram in Figure 1.2 and calculate its average value \bar{i}_L .

1.5 Now the buck converter should operate in boundary conduction mode (BCM). Determine the minimum possible duty cycle D_{BCM} that has to be adjusted for that operation status. How large are the current ripple Δi_L and the average current \bar{i}_L ?

1.6 Insert the known points of DCM and BCM in the diagram in Figure 1.3 and sketch roughly the characteristic of the average inductor current \bar{i}_L over the duty cycle D .

Figure 1.3: Average inductor current \bar{i}_L over duty cycle D

Task 2: Boost Converter**(20 Credits)**

A boost converter is used in a PC for voltage adjustment. The circuit diagram of the converter is shown in Figure 2.1.

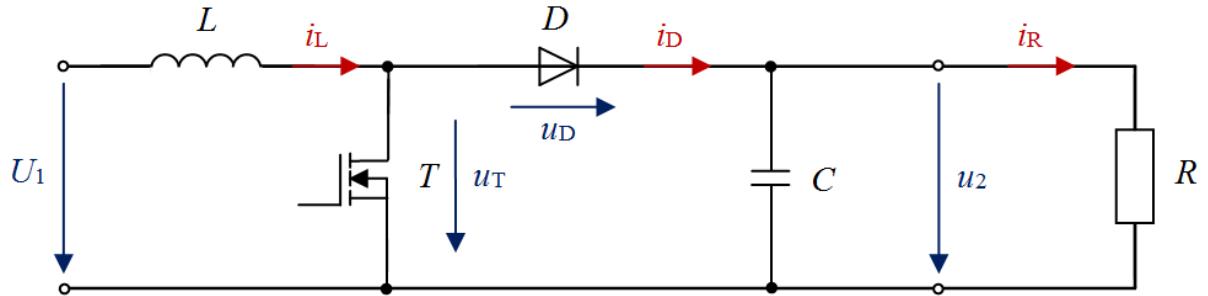


Figure 2.1: Topology of boost converter

The following values of the converter are known:

- Input voltage: $U_1 = 5 \text{ V}$
- Output voltage: $\bar{u}_2 = 12 \text{ V}$
- Inductance: $L = 100 \mu\text{H}$
- Switching frequency: $f_s = 100 \text{ kHz}$

The boost converter can be considered as ideal and operates in steady state. The input DC voltage U_1 is assumed constant. The output voltage ripple Δu_2 can be neglected.

- 2.1** Derive the ideal control characteristic $U_2/U_1 = f(D)$ of the boost converter. Set up the equations for $i_L(t = DT_s)$ and $i_L(t = T_s)$ as functions of U_1, U_2, D and T_s in order to solve for the asked voltage ratio U_2/U_1 . Assume that the boost converter operates in continuous conduction mode (CCM). Calculate the duty cycle D .
- 2.2** Determine the current ripple Δi_L of the inductor current $i_L(t)$.
- 2.3** The boost converter operates in CCM. The load resistor R should now be adjusted so that the converter operates in boundary conduction mode (BCM). Determine the required resistance value R_{BCM} .

Now the load resistor is further increased ($R > R_{\text{BCM}}$) to force the converter to operate in discontinuous conduction mode (DCM). In order to keep the output voltage constant the duty cycle is changed to $D = 0.35$.

- 2.4** Draw the transistor voltage $u_T(t)$, the inductor current $i_L(t)$ and the diode current $i_D(t)$ over two periods in the diagrams in Figure 2.2. Determine the current conducting time span Δt_D of the diode (after the transistor has been switched off).

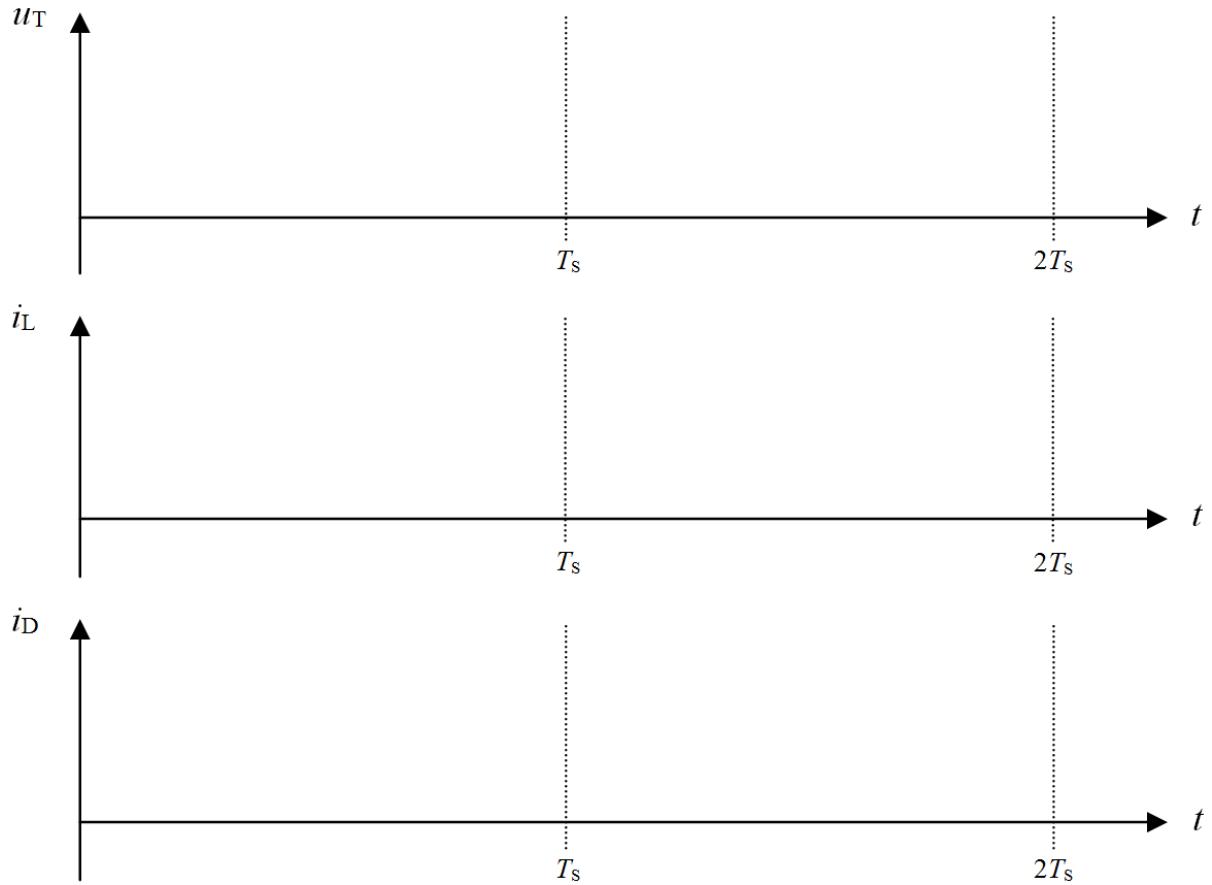


Figure 2.2: Transistor voltage, inductor and diode current

- 2.5 Calculate the average values \bar{i}_L and \bar{i}_D of the inductor current $i_L(t)$ and the diode current $i_D(t)$. Determine the value of the load resistance R .

Task 3: Four-Quadrant Converter**(20 Credits)**

A suburban train is equipped with a four-quadrant converter (4QC) to rectify the AC line voltage and to provide a nearly constant DC voltage for the traction drives and the on-board electrical systems. The system is shown in Figure 3.1:

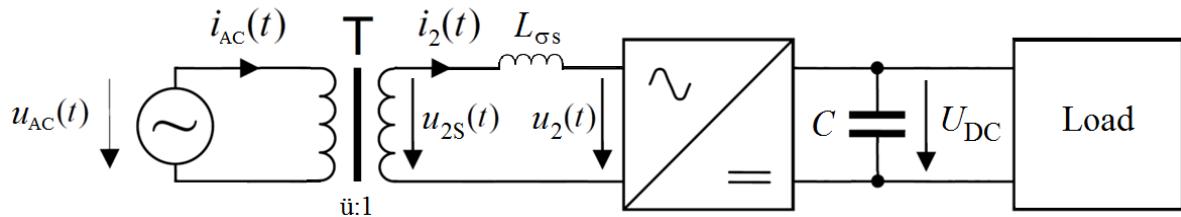


Figure 3.1: Transformer, four-quadrant converter and DC-link between grid and load

The following physical quantities are known:

- Grid voltage: $U_{AC} = 15 \text{ kV}$
- Grid frequency: $f = 16.667 \text{ Hz}$
- Secondary-side leakage inductance: $L_{\sigma s} = 6.25 \text{ mH}$
- DC-link voltage: $U_{DC} = 2.5 \text{ kV}$

The four-quadrant converter can be considered as ideal (no losses, no interlocking times). The primary leakage inductance and the winding resistances of the transformer can be neglected. In the above diagram $L_{\sigma s}$ is the transformer's leakage inductance. So, the transformer circuit symbol is considered as ideal transformer.

- 3.1** Draw the structure of the PWM unit for interleaved switching mode in Figure 3.2. Input is the normalized reference voltage, output are the two switching signals for the two legs of the 4QC.

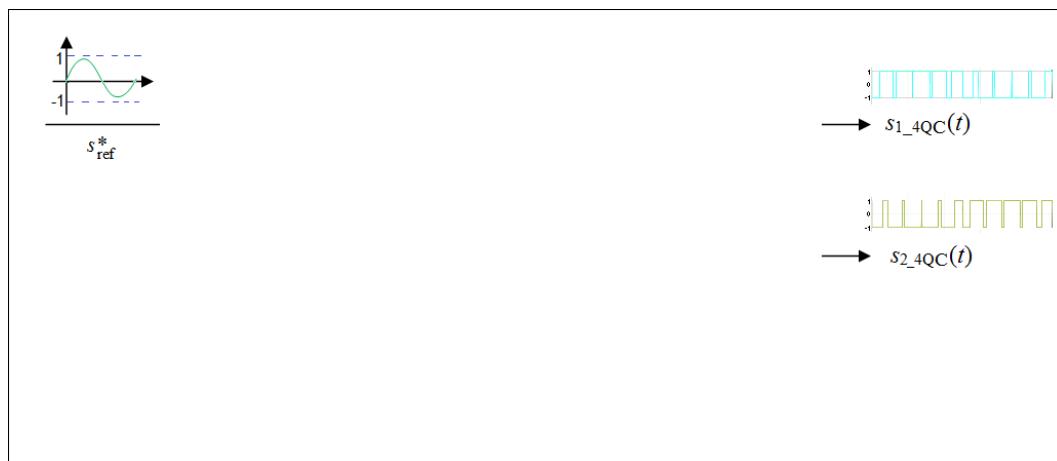


Figure 3.2: PWM unit for interleaved switching mode

- 3.2** What value for the switching frequency f_S has to be selected so that the maximum occurring inductor current ripple $\Delta i_{2\max}$ does not exceed 100 A for interleaved switching mode? Determine the frequency of the current ripple that results from interleaved switching mode. For which values of $s_{ref}^*(t)$ does the maximum current ripple appear?

In Figure 3.3 the root mean square value of the secondary current I_2 and the power P drawn from the grid were recorded during the ride of the train from one station to the next. The ride required a total amount of energy of $W = 3.5 \text{ kWh}$.

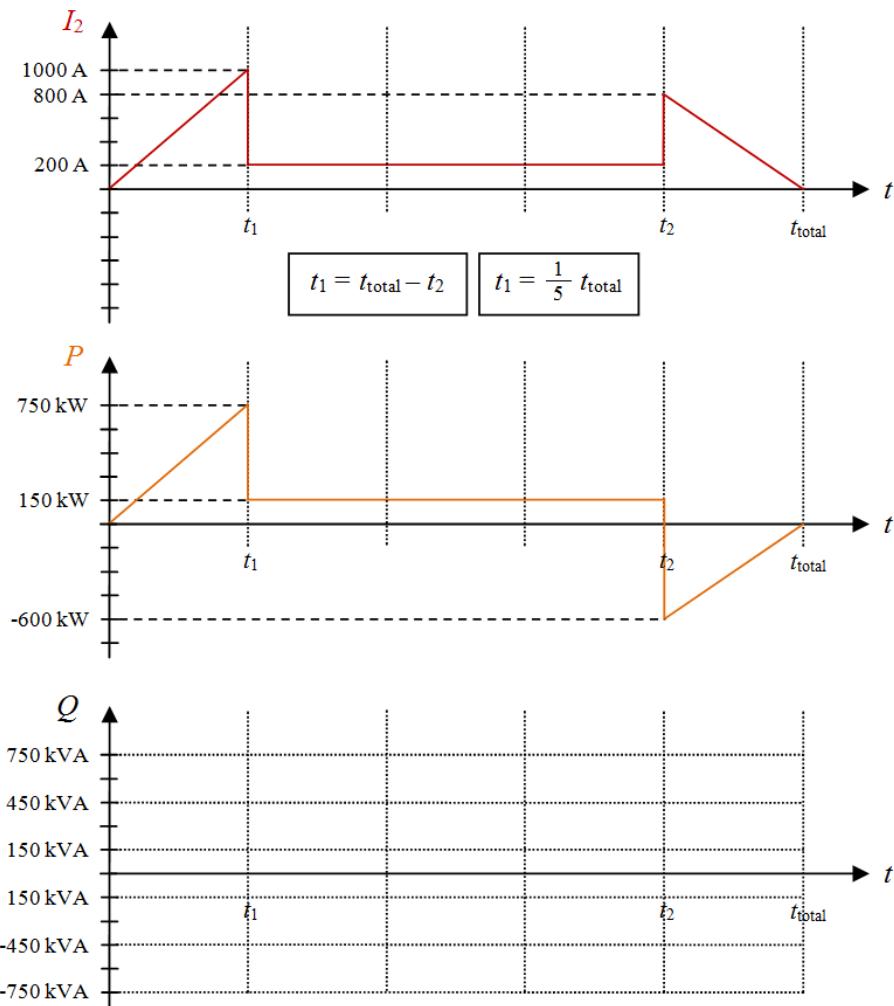


Figure 3.3: Diagrams showing secondary side current (top), active power (centre) and reactive power (bottom)

- 3.3 Calculate the transformer turns ratio $\tilde{\nu}$ with help of the given diagrams above. Determine the duration t_{total} of the train ride.
- 3.4 Draw the grid-side phasor diagrams for the times $t = t_1$ (end of acceleration phase), $t_1 < t < t_2$ (constant power phase) and $t = t_2$ (start of the decelerating phase). Calculate the phase angles φ (angle between grid voltage and transformed converter voltage) for the three cases. It can be assumed that the controller operates in power factor correction mode ($\cos(\varphi_{ui}) = 1$) leading solely to active power drawn from or fed into the grid. Use the following scales: $10 \text{ kV} \doteq 5 \text{ cm}$; $100 \text{ A} \doteq 5 \text{ cm}$.
- 3.5 Calculate the reactive power Q which is required by the transformer and delivered by the 4QC for the three cases of subtask 3.4), insert the calculated values in the power diagram in Figure 3.3 and sketch roughly its course.

Task 4: Line-Commutated Rectifier

(20 Credits)

A 3-pulse thyristor-controlled converter is connected to the 3-phase grid via a Dy-transformer configuration and supplies a resistive load. The complete configuration is shown in Figure 4.1:

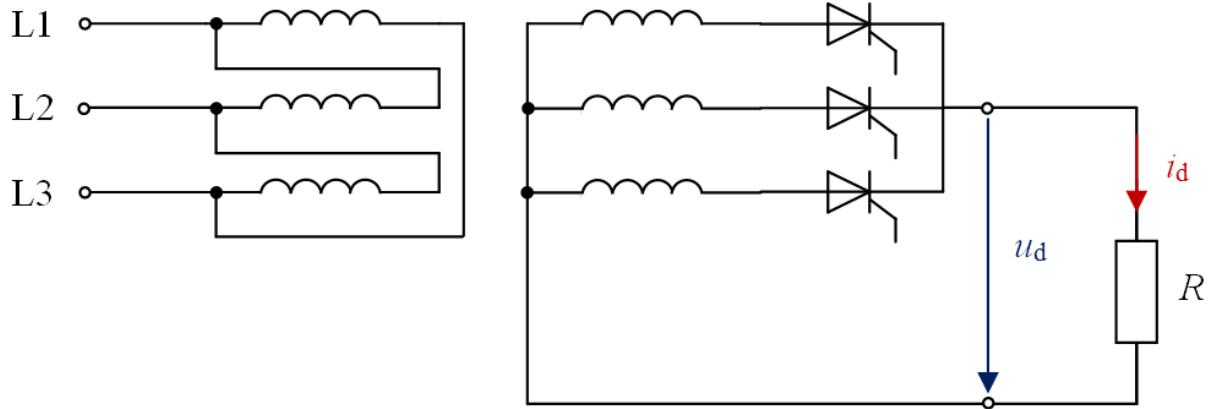


Figure 4.1: Line commutated rectifier in M3 configuration

The transformer and converter can be considered as ideal (no losses, no magnetic leakage).

- 4.1** Sketch the output voltage $u_d(t)$ in the diagrams in Figure 4.2 for the control angles $\alpha = 0^\circ$, 45° and 135° . What is the control angle limit value α_L that represents the boundary control angle between CCM and DCM operation?

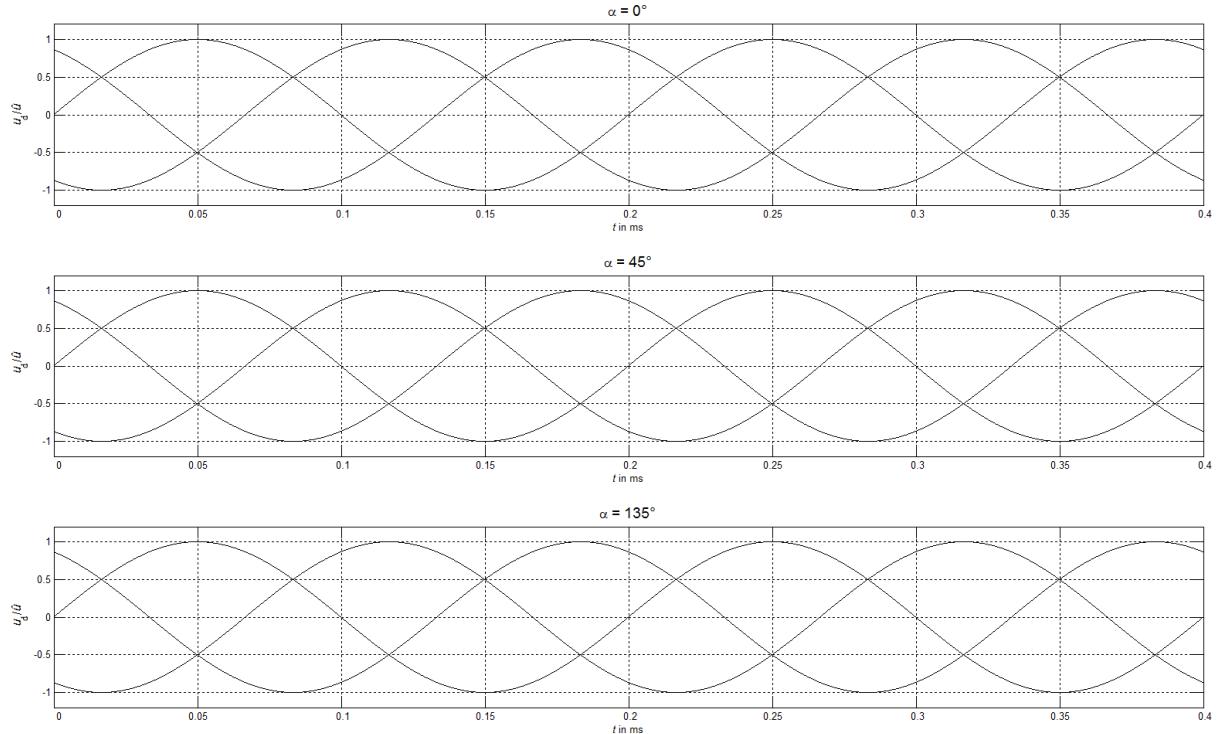


Figure 4.2: Normalized output voltage u_d/\hat{u} for different control angles

- 4.2** Calculate and draw in Figure 4.3 the control characteristic $U_d/U_{d\max} = f(\alpha)$ of the converter for the continuous conduction mode ($\alpha < \alpha_L$) and discontinuous conduction mode ($\alpha > \alpha_L$) range.

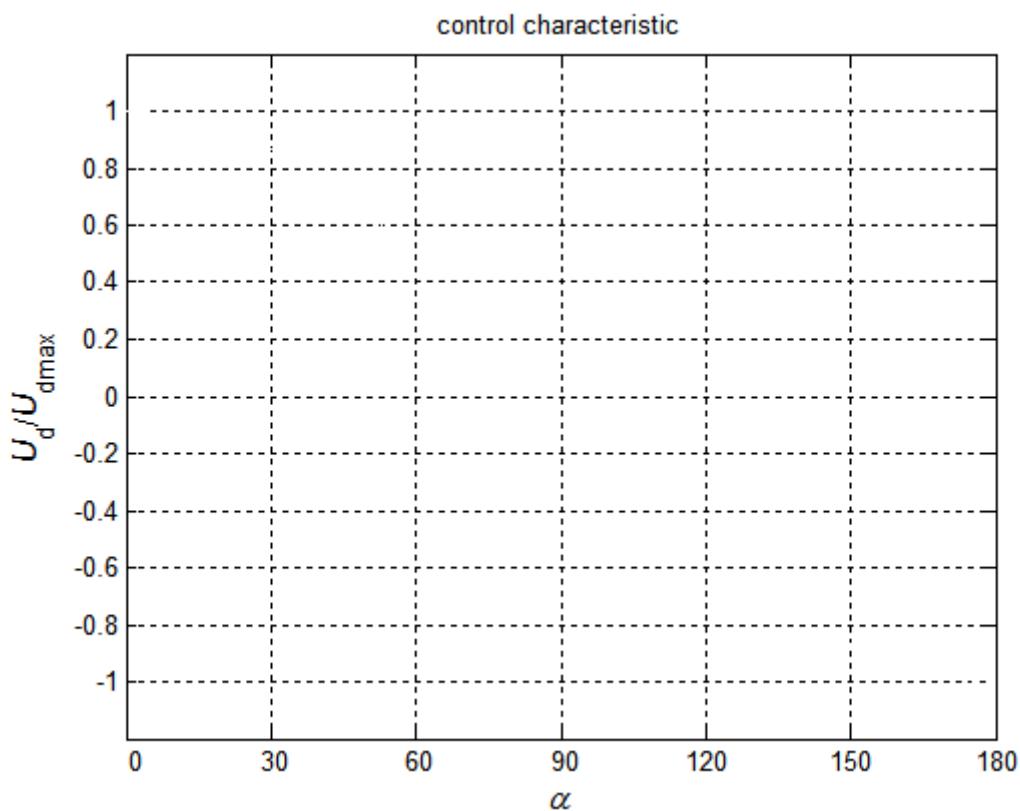


Figure 4.3: Control characteristic of converter for CCM and DCM

- 4.3 Determine the active power P_R which is consumed by the load resistance R as a function of the control angle α for the continuous conduction mode (CCM).
- 4.4 The converter supplies a resistive load. Describe in full sentences why reactive power appears on the mains side as well? Assign the different types of losses to its causes.

The following mathematical relations might be needed for some of the subtasks:

$$\sin(x \pm y) = \sin(x) \cdot \cos(y) \pm \cos(x) \cdot \sin(y)$$

$$\cos(x \pm y) = \cos(x) \cdot \cos(y) \mp \sin(x) \cdot \sin(y)$$

$$\int \cos^2(ax) dx = \frac{x}{2} + \frac{1}{4a} \sin(2ax)$$

$$\int \sin^2(ax) dx = \frac{x}{2} - \frac{1}{4a} \sin(2ax)$$

Solution**Task 1) Buck Converter****[20 Credits]**

- 1.1) Average value of input current and power drawn from the DC source:

$$2 \quad \bar{i}_1 = \frac{\frac{1}{2} \cdot D_{DCM} \cdot T_S \cdot i_{1\max}}{T_S} = \frac{1}{2} \cdot D_{DCM} \cdot i_{1\max} = \frac{1}{2} \cdot 0,2 \cdot 0,876 \text{ A} = \underline{\underline{0,0876 \text{ A}}}$$

$$1 \quad P = U_1 \cdot \bar{i}_1 = 12 \text{ V} \cdot 0,0876 \text{ A} = \underline{\underline{1,0512 \text{ W}}}$$

[3 Credits]

- 1.2) Output voltage and output current:

$$1 \quad U_2 = \sqrt{P \cdot R} = \sqrt{1,0512 \text{ W} \cdot 10 \Omega} = \underline{\underline{3,2422 \text{ V}}}$$

$$1 \quad I_2 = \frac{U_2}{R} = \frac{3,2422 \text{ V}}{10 \Omega} = \underline{\underline{0,324 \text{ A}}}$$

[2 Credits]

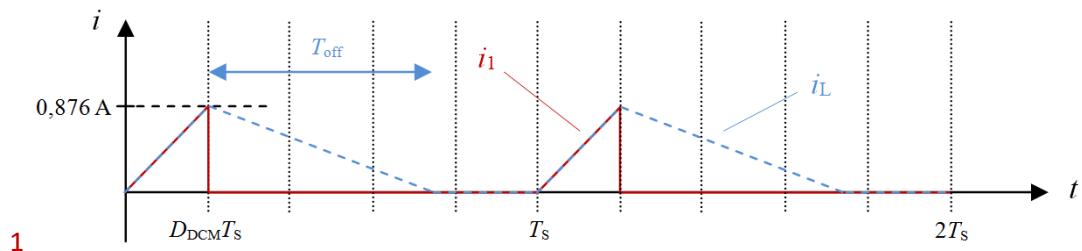
- 1.3) Value of inductance:

$$1+1 \quad \Delta i_L = \frac{U_1 - U_2}{L} D_{DCM} T_S \rightarrow L = \frac{U_1 - U_2}{\Delta i_L} D_{DCM} T_S = \frac{12 \text{ V} - 3,24 \text{ V}}{0,876 \text{ A}} \cdot 0,2 \cdot 0,00002 \text{ s} = \underline{\underline{40 \mu\text{H}}}$$

[2 Credits]

- 1.4) Off-time, inductor current and average value of inductor current:

$$1+1 \quad i_{L\min} = i_{L\max} - \frac{U_2}{L} T_{off} = 0 \rightarrow T_{off} = \frac{L i_{L\max}}{U_2} = \frac{0,00004 \text{ H} \cdot 0,876 \text{ A}}{3,24 \text{ V}} = \underline{\underline{10,812 \mu\text{s}}}$$



$$1 \quad \bar{i}_L = \frac{\frac{1}{2} \cdot (D_{DCM} \cdot T_S + T_{off}) \cdot i_{1\max}}{T_S} = \frac{\frac{1}{2} \cdot (0,2 \cdot 20 \mu\text{s} + 10,812 \mu\text{s}) \cdot 0,876 \text{ A}}{20 \mu\text{s}} = \underline{\underline{0,324 \text{ A}}} = I_2$$

[4 Credits]

- 1.5) Duty cycle for BCM, inductor current ripple and average inductor current:

In BCM, value of average inductor current is half of the inductor current ripple:

$$1 \quad \bar{i}_L = \frac{\Delta i_L}{2} = \frac{D(1-D)U_1 T_S}{2L}$$

$$1 \quad \bar{i}_L = \frac{U_2}{R} = \frac{DU_1}{R}$$

Equating the two equations gives:

$$\frac{D(1-D)U_1 T_S}{2L} = \frac{DU_1}{R}$$

$$1 \quad \frac{(1-D)T_S}{2L} = \frac{1}{R}$$

$$1 - D = \frac{2L}{T_S R}$$

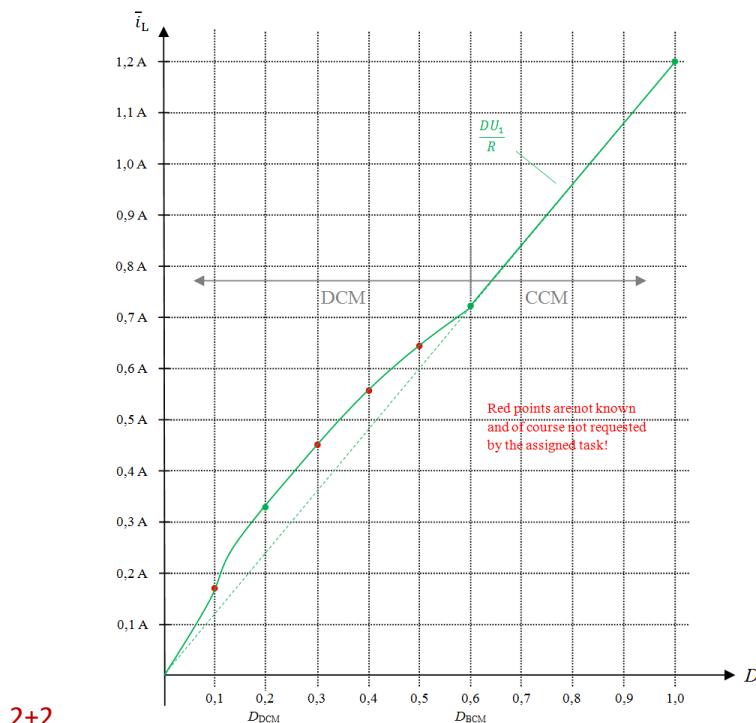
$$1 \quad D = 1 - \frac{2L}{T_S R} = 1 - \frac{2 \cdot 40 \mu\text{H}}{20 \mu\text{s} \cdot 10 \Omega} = 1 - 0.4 = \underline{\underline{0.6}}$$

$$\Delta i_L = \frac{D(1-D)U_1 T_S}{L} = \frac{0.6(1-0.6) \cdot 12 \text{ V} \cdot 20 \mu\text{s}}{40 \mu\text{H}} = \underline{\underline{1.44 \text{ A}}}$$

$$1 \quad \bar{i}_L = \frac{\Delta i_L}{2} = \frac{1.44 \text{ A}}{2} = \underline{\underline{0.72 \text{ A}}}$$

[5 Credits]

1.6) Average inductor current over duty cycle:



[4 Credits]

Task 2)

Boost Converter

[20 Credits]

2.1) Control characteristic and duty cycle:

$$\frac{U_2}{U_1} = f(D) = ?$$

Setting up the two formulas for the inductor current (steady state is assumed):

$$1 \quad i_L(t = DT_S) = \frac{U_1}{L} DT_S + i_L(t = 0)$$

$$1 \quad i_L(T_S) = i_L(t = DT_S) + \frac{U_1 - U_2}{L} (1 - D) T_S = i_L(t = 0)$$

Solving for the searched voltage ratio:

$$\frac{U_1}{L} DT_S + i_L(t = 0) + \frac{U_1 - U_2}{L} (1 - D) T_S = i_L(t = 0)$$

$$\frac{U_1}{L} DT_S + \frac{U_1 - U_2}{L} (1 - D) T_S = 0$$

$$\frac{U_1}{L} D + \frac{U_1 - U_2}{L} (1 - D) = 0$$

$$2 \quad U_1 D + (U_1 - U_2)(1 - D) = 0$$

$$U_1 D + U_1 - U_2 - U_1 D + U_2 D = 0$$

$$U_1 - U_2 + U_2 D = 0$$

$$-U_2 + U_2 D = -U_1$$

$$U_2 - U_2 D = U_1$$

$$U_2(1 - D) = U_1$$

$$1 \quad \frac{U_2}{U_1} = \frac{1}{\underline{\underline{1-D}}}$$

Adjusted duty cycle:

$$1 - D = \frac{U_1}{U_2}$$

$$1 \quad D = 1 - \frac{U_1}{U_2} = \frac{U_2 - U_1}{U_2} = \frac{12 \text{ V} - 5 \text{ V}}{12 \text{ V}} = \underline{\underline{0.583}}$$

[6 Credits]

2.2) Inductor current ripple:

$$1 \quad \Delta i_L = \frac{U_1}{L} D T_S = \frac{5 \text{ V}}{100 \mu\text{H}} 0.583 \cdot 10 \mu\text{s} = \underline{\underline{0.2915 \text{ A}}}$$

[1 Credit]

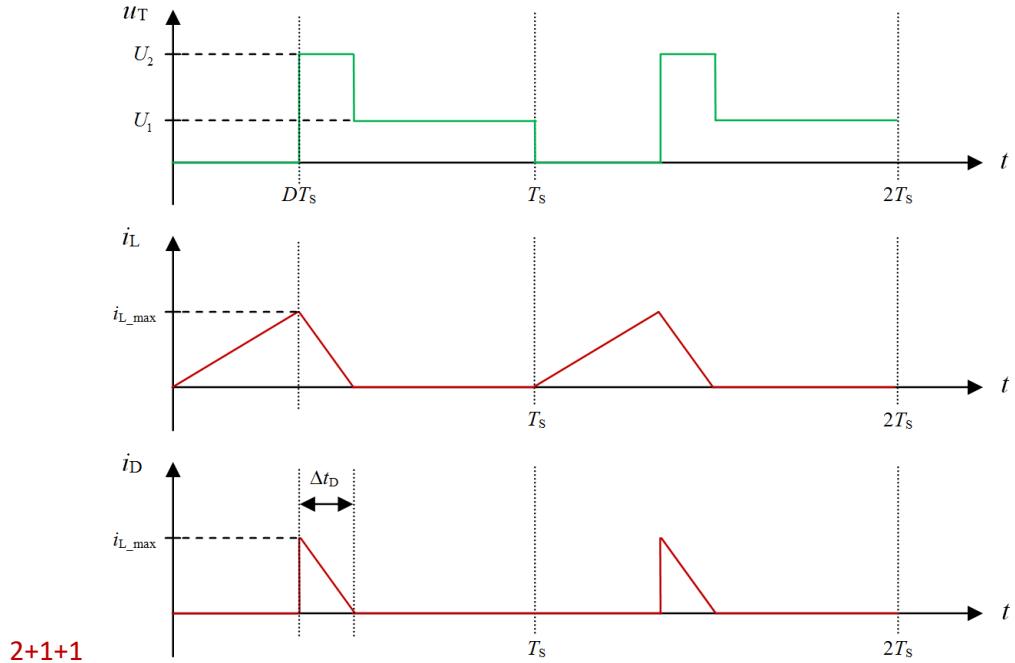
2.3) Resistance value for boundary conduction mode:

$$2 \quad \bar{i}_D = \bar{i}_R = \frac{U_2}{R_{\max}} = \bar{i}_L(1 - D) = \frac{\Delta i_L}{2}(1 - D)$$

$$1 \quad \frac{U_2}{R_{\text{BCM}}} = \frac{\Delta i_L}{2}(1 - D) \rightarrow R_{\text{BCM}} = \frac{2U_2}{\Delta i_L(1-D)} = \frac{2 \cdot 12 \text{ V}}{0.2915 \text{ A} \cdot (1-0.583)} = \underline{\underline{197.44 \Omega}}$$

[3 Credits]

2.4) Current conduction time span of diode and diagrams:



$$1 \quad i_{L_{\max}} = \frac{U_1}{L} D T_S = \frac{5 \text{ V}}{100 \mu\text{H}} 0.35 \cdot 10 \mu\text{s} = \underline{\underline{0.175 \text{ A}}}$$

$$1 \quad i_L(t = t_{\text{Lück}}) = \frac{U_1 - U_2}{L} \Delta t_D + i_{L_{\max}} = 0 \text{ A}$$

$$1 \quad \Delta t_D = \frac{(0 \text{ A} - i_{L_{\max}})L}{U_1 - U_2} = \frac{L \cdot i_{L_{\max}}}{-U_1 + U_2} = \frac{100 \mu\text{H} \cdot 0.175 \text{ A}}{-5 \text{ V} + 12 \text{ V}} = \underline{\underline{2.5 \mu\text{s}}}$$

[7 Credits]

2.5) Average value of inductor and diode current and value of resistor:

$$\textcolor{red}{1} \quad \bar{i}_L = \frac{\frac{1}{2}i_{L\max} \cdot 0.6T_S}{T_S} = \frac{1}{4}i_{L\max} = \underline{\underline{0.0525 \text{ A}}}$$

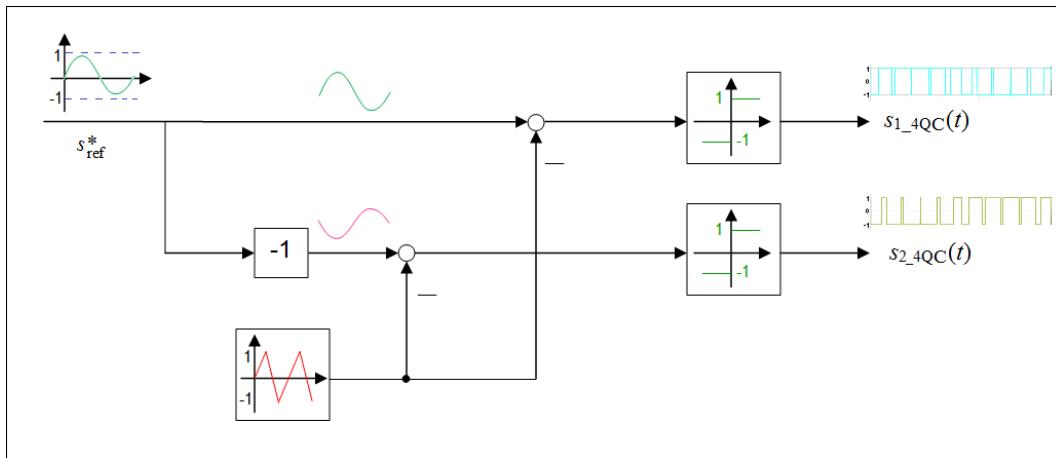
$$\textcolor{red}{1} \quad \bar{i}_D = \frac{\frac{1}{2}i_{L\max} \cdot 0.25T_S}{T_S} = \frac{1}{8}i_{L\max} = \underline{\underline{0.02187 \text{ A}}}$$

$$\textcolor{red}{1} \quad R = \frac{U_2}{\bar{i}_R} = \frac{U_2}{\bar{i}_D} = \frac{12 \text{ V}}{0.02187 \text{ A}} = \underline{\underline{548.57 \Omega}}$$

[3 Credits]

Task 3)**Four-Quadrant Converter****[20 Credits]**

- 3.1) PWM unit structure for interleaved operation mode of 4QC:

**[3 Credits]**

- 3.2) Leakage inductance for requested maximum current ripple:

$$1 \quad \Delta i_{2,\text{INTERLEAVED}} = |s^*|(1 - |s^*|) \cdot \frac{U_{\text{DC}}}{2f_S \cdot L_{\sigma s}}$$

Maximum current ripple for $s^*_{\text{ref}}(t) = \pm 0.5$: 1

$$1 \quad \Delta i_{2,\text{max_INTERLEAVED}} = \frac{U_{\text{DC}}}{8f_S \cdot L_{\sigma s}} \rightarrow f_S = \frac{U_{\text{DC}}}{\Delta i_{2,\text{max_IL}} \cdot 8 \cdot L_{\sigma s}} = \frac{2.5 \text{ kV}}{100 \text{ A} \cdot 8 \cdot 6.25 \text{ mH}} = \underline{\underline{500 \text{ Hz}}}$$

Pulse frequency:

$$1 \quad f_p = 2 \cdot f_S = 2 \cdot 500 \text{ Hz} = \underline{\underline{1000 \text{ Hz}}}$$

[4 Credits]

- 3.3) Transformer turns ratio and duration of train ride:

$$1 \quad \ddot{u} = \frac{I_2}{I_{\text{AC}}} = \frac{I_2}{P/U_{\text{AC}}} = \frac{1000 \text{ A}}{750 \text{ kW}/15 \text{ kV}} = \underline{\underline{20}}$$

$$1 \quad W = \frac{1}{2} \cdot P(t = t_1) \cdot t_1 + P(t_1 < t < t_2) \cdot (t_2 - t_1) + \frac{1}{2} \cdot P(t = t_2) \cdot (t_{\text{total}} - t_2)$$

$$W = \frac{1}{2} \cdot P(t = t_1) \cdot t_1 + P(t_1 < t < t_2) \cdot 3t_1 + \frac{1}{2} \cdot P(t = t_2) \cdot t_1$$

$$W = \left[\frac{1}{2} \cdot P(t = t_1) + P(t_1 < t < t_2) \cdot 3 + \frac{1}{2} \cdot P(t = t_2) \right] t_1$$

$$1 \quad t_1 = \frac{W}{\frac{1}{2}P(t=t_1)+P(t_1 < t < t_2) \cdot 3 + \frac{1}{2}P(t=t_2)} = \frac{3.5 \text{ kWh}}{\frac{1}{2} \cdot 750 \text{ kW} + 150 \text{ kW} \cdot 3 - \frac{1}{2} \cdot 600 \text{ kW}}$$

$$t_1 = \frac{3.5 \text{ kWh} \cdot \frac{3600 \text{ s}}{\text{h}}}{525 \text{ kW}} = 24 \text{ s}$$

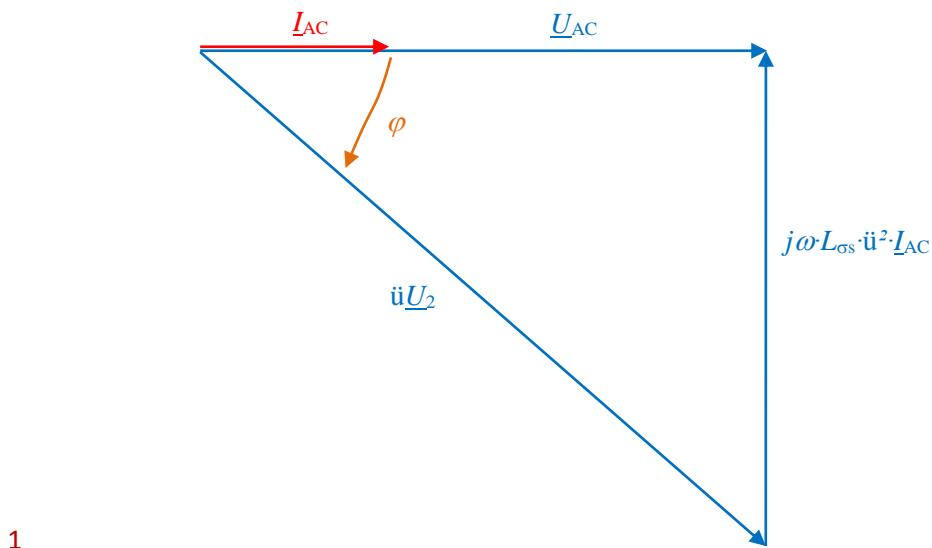
$$1 \quad t_{\text{total}} = 5t_1 = \underline{\underline{120 \text{ s}}}$$

[4 Credits]

3.4) Grid-side phasor diagrams for three cases:

Case 1: End of acceleration phase ($t = t_1$)

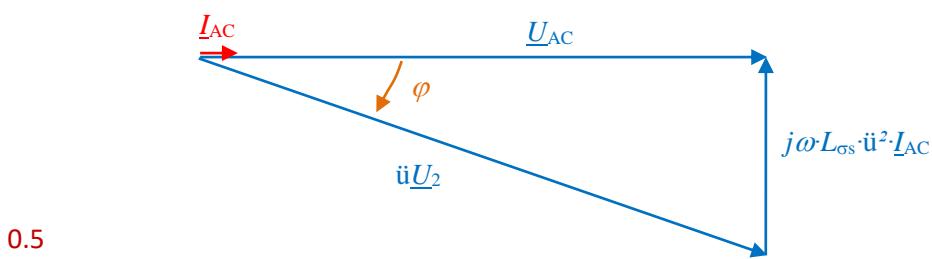
$$1 \quad P = U_{\text{AC}} \cdot I_{\text{AC}} \cdot \cos(\varphi_{ui}) \rightarrow I_{\text{AC}} = \frac{750 \text{ kW}}{15 \text{ kV} \cdot \cos(0^\circ)} = \underline{\underline{50 \text{ A}}}$$



$$1 \quad \varphi = \arctan\left(\frac{\omega \cdot L_{\text{gs}} \cdot \ddot{u}^2 \cdot I_{\text{AC}}}{U_{\text{AC}}}\right) = \arctan\left(\frac{2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot 20^2 \cdot 50 \text{ A}}{15 \text{ kV}}\right) = \underline{\underline{41.11^\circ}}$$

Case 2: Constant power phase ($t_1 < t < t_2$)

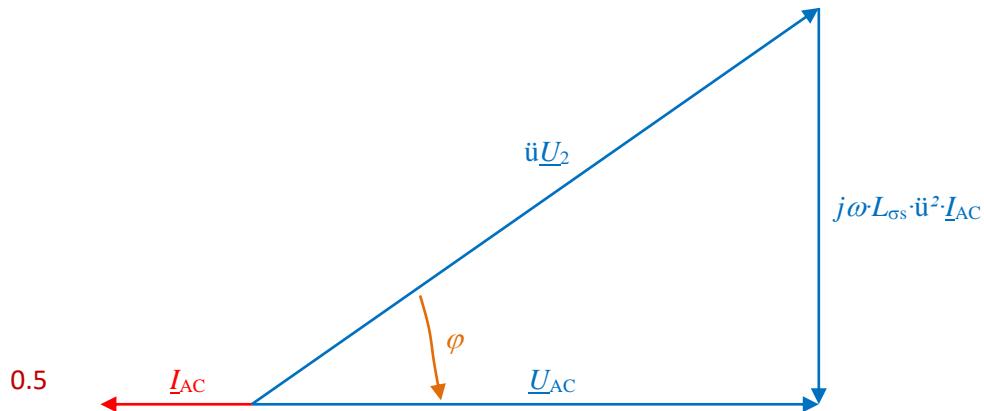
$$0.5 \quad P = U_{\text{AC}} \cdot I_{\text{AC}} \cdot \cos(\varphi_{ui}) \rightarrow I_{\text{AC}} = \frac{150 \text{ kW}}{15 \text{ kV} \cdot \cos(0^\circ)} = \underline{\underline{10 \text{ A}}}$$



$$0.5 \quad \varphi = \arctan\left(\frac{\omega \cdot L_{\text{gs}} \cdot \ddot{u}^2 \cdot I_{\text{AC}}}{U_{\text{AC}}}\right) = \arctan\left(\frac{2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot 20^2 \cdot 10 \text{ A}}{15 \text{ kV}}\right) = \underline{\underline{9.90^\circ}}$$

Case 3: Start of deceleration phase ($t = t_2$)

0.5 $P = U_{AC} \cdot I_{AC} \cdot \cos(\varphi_{ui}) \rightarrow I_{AC} = \frac{-600 \text{ kW}}{15 \text{ kV} \cdot \cos(-180^\circ)} = \underline{\underline{40 \text{ A}}}$



0.5 $\varphi = -\arctan\left(\frac{\omega \cdot L_{\sigma s} \cdot \ddot{u}^2 \cdot I_{AC}}{U_{AC}}\right) = -\arctan\left(\frac{2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot 20^2 \cdot 40 \text{ A}}{15 \text{ kV}}\right) = \underline{\underline{-34.92^\circ}}$

[6 Credits]

3.5) Reactive Power (required by transformer):

Case 1: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_2)^2 = 2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot (1000 \text{ A})^2$

1 $Q = \underline{\underline{654.50 \text{ kVA}}}$

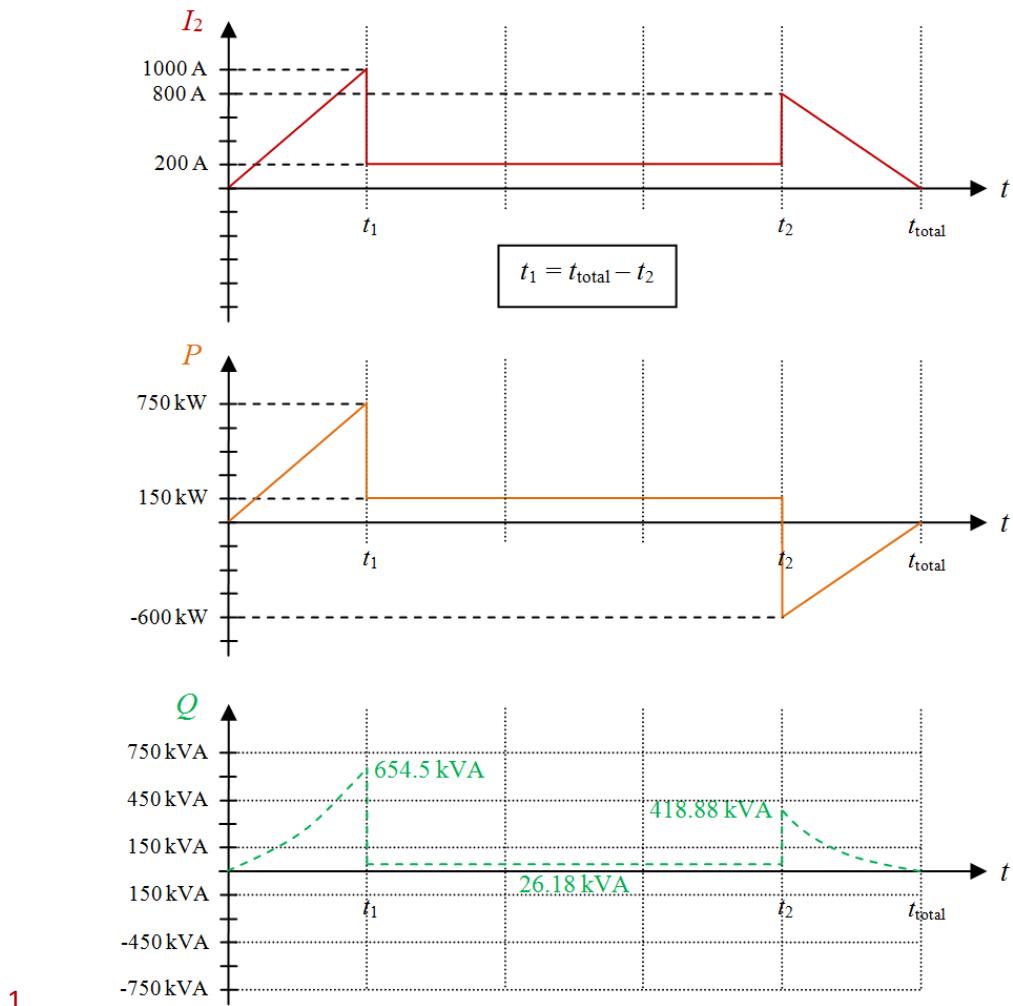
Case 2: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_2)^2 = 2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot (200 \text{ A})^2$

0.5 $Q = \underline{\underline{26.18 \text{ kVA}}}$

Case 3: $Q = 2\pi \cdot f \cdot L_{\sigma s} \cdot (I_2)^2 = 2\pi \cdot 16.667 \text{ Hz} \cdot 0.00625 \text{ H} \cdot (800 \text{ A})^2$

0.5 $Q = \underline{\underline{418.88 \text{ kVA}}}$

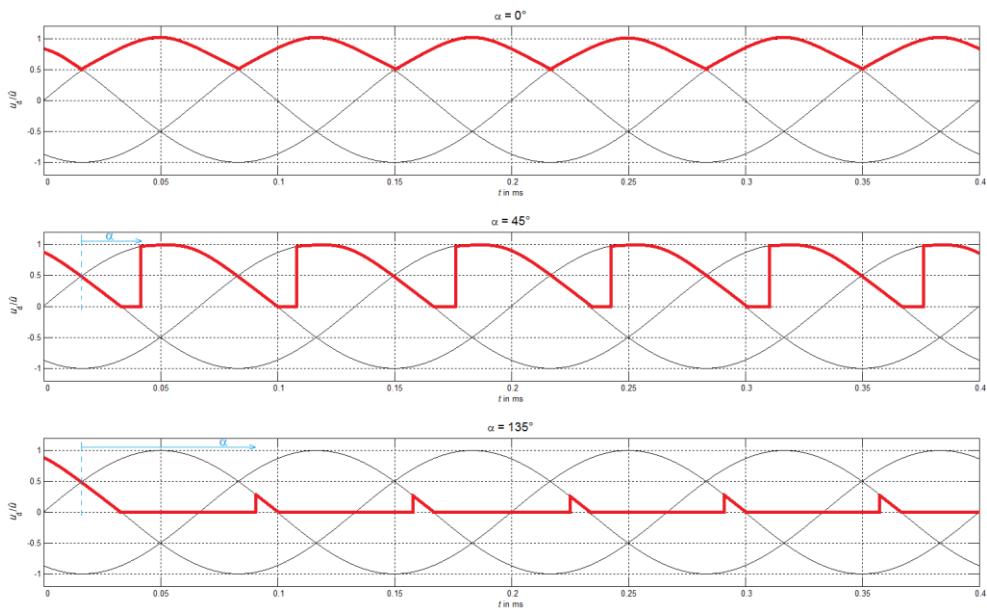
Reactive power has to be provided by the 4QC. The reactive power oscillates between transformer and 4QC.



[3 Credits]

Task 4)**Line-Commutated Rectifier****[20 Credits]**

4.1) Output voltage waveforms for three different control angles:

**[4 Credits]**

4.2) Control characteristic of line-commutated rectifier:

CCM ($0 \leq \alpha < 30^\circ$):

$$1 \quad U_d = \frac{p}{2\pi} \int_{30^\circ + \alpha}^{150^\circ + \alpha} \hat{u} \cdot \sin(\omega t) d\omega t = \frac{3}{2\pi} \hat{u} [-\cos(\omega t)]_{30^\circ + \alpha}^{150^\circ + \alpha}$$

$$U_d = \frac{3}{2\pi} \hat{u} [-\cos(150^\circ + \alpha) + \cos(30^\circ + \alpha)]$$

$$1 \quad U_d = \frac{3}{2\pi} \hat{u} [-\cos(150^\circ) \cos(\alpha) + \sin(150^\circ) \sin(\alpha) + \cos(30^\circ) \cos(\alpha) - \sin(30^\circ) \sin(\alpha)]$$

$$U_d = \frac{3}{2\pi} \hat{u} [-\cos(150^\circ) \cos(\alpha) + \cos(30^\circ) \cos(\alpha)]$$

$$U_d = \frac{3}{2\pi} \hat{u} \left[\frac{\sqrt{3}}{2} \cos(\alpha) + \frac{\sqrt{3}}{2} \cos(\alpha) \right] = \frac{3\sqrt{3}}{2\pi} \hat{u} \cos(\alpha)$$

$$1 \quad U_{d_{\max}}|_{\alpha=0^\circ} = \frac{3\sqrt{3}}{2\pi} \hat{u}$$

$$1 \quad f_{CCM}(\alpha) = \frac{U_d}{U_{d_{\max}}} = \frac{\frac{3\sqrt{3}}{2\pi} \hat{u} \cos(\alpha)}{\frac{3\sqrt{3}}{2\pi} \hat{u}} = \underline{\underline{\cos(\alpha)}}$$

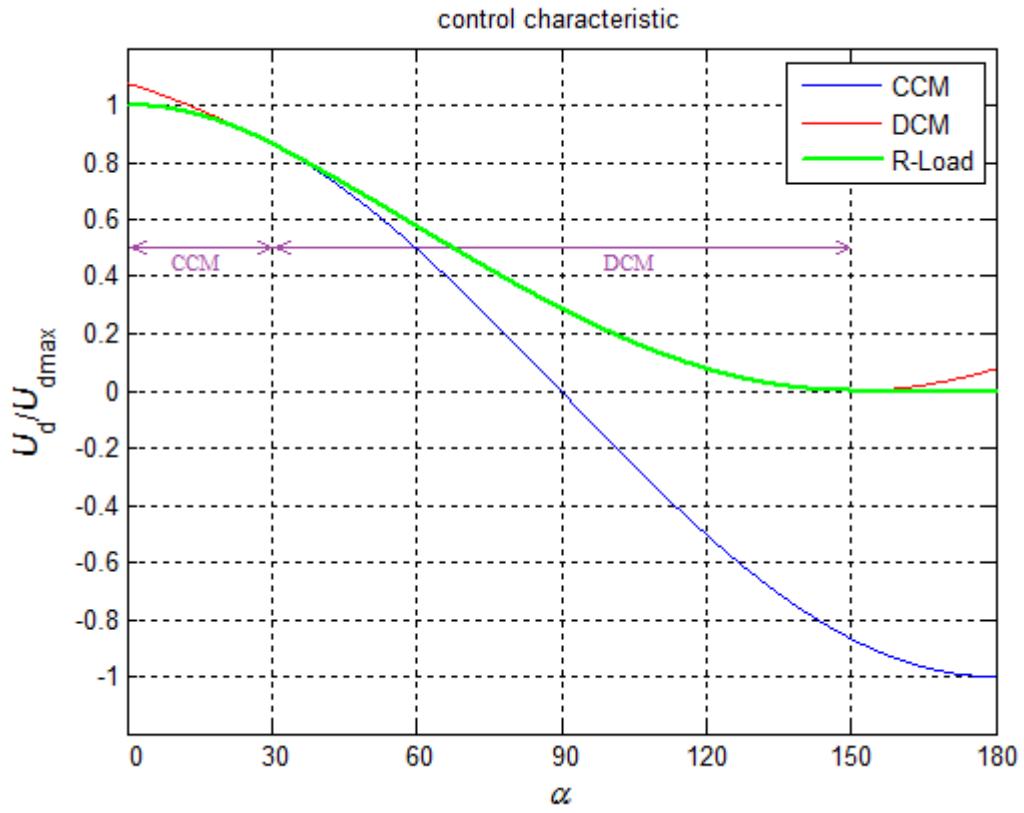
DCM ($30^\circ \leq \alpha < 150^\circ$):

$$\begin{aligned}
 1 \quad U_d &= \frac{p}{2\pi} \int_{30^\circ+\alpha}^{180^\circ} \hat{u} \cdot \sin(\omega t) d\omega t = \frac{3}{2\pi} \hat{u} [-\cos(\omega t)]_{30^\circ+\alpha}^{180^\circ} \\
 &= \frac{3}{2\pi} \hat{u} [-\cos(180^\circ) + \cos(30^\circ + \alpha)] \\
 &= \frac{3}{2\pi} \hat{u} [-\cos(180^\circ) + \cos(30^\circ) \cos(\alpha) - \sin(30^\circ) \sin(\alpha)] \\
 1 \quad U_d &= \frac{3}{2\pi} \hat{u} \left[1 + \frac{\sqrt{3}}{2} \cos(\alpha) - \frac{1}{2} \sin(\alpha) \right]
 \end{aligned}$$

$$U_{d_{\max}}|_{\alpha=0^\circ} = \frac{3\sqrt{3}}{2\pi} \hat{u} \quad (\text{Universal reference value for CCM and DCM})$$

$$1 \quad f_{DCM}(\alpha) = \frac{U_d}{U_{d_{\max}}} = \frac{\frac{3}{2\pi} \hat{u} \left[1 + \frac{\sqrt{3}}{2} \cos(\alpha) - \frac{1}{2} \sin(\alpha) \right]}{\frac{3\sqrt{3}}{2\pi} \hat{u}} = \underline{\underline{\frac{1}{\sqrt{3}} + \frac{1}{2} \cos(\alpha) - \frac{1}{2\sqrt{3}} \sin(\alpha)}} = \frac{1}{\sqrt{3}} [1 + \cos(\alpha + 30^\circ)]$$

Control characteristic of converter for R-load:



1+1+1

[10 Credits]

4.3) Active power in dependency of control angle:

CCM ($0 \leq \alpha < 30^\circ$):

$$\begin{aligned}
 1 \quad P_R &= \frac{U_{d,RMS}^2}{R} = \frac{\frac{p}{2\pi} \int_{30^\circ+\alpha}^{150^\circ+\alpha} \hat{u}^2 \cdot \sin^2(\omega t) d\omega t}{R} = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \int_{30^\circ+\alpha}^{150^\circ+\alpha} \sin^2(\omega t) d\omega t \\
 &P_R = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{\omega t}{2} - \frac{\sin(2\omega t)}{4} \right]_{30^\circ+\alpha}^{150^\circ+\alpha} = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{150^\circ+\alpha}{2} - \frac{\sin(2 \cdot 150^\circ + 2\alpha)}{4} - \frac{30^\circ+\alpha}{2} + \frac{\sin(2 \cdot 30^\circ + 2\alpha)}{4} \right] \\
 2 \quad P_R &= \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{\frac{5\pi}{6}+\alpha}{2} - \frac{\sin(300^\circ+2\alpha)}{4} - \frac{\frac{\pi}{6}+\alpha}{2} + \frac{\sin(60^\circ+2\alpha)}{4} \right] \\
 &P_R = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{5\pi}{12} - \frac{\sin(300^\circ) \cdot \cos(2\alpha) + \cos(300^\circ) \cdot \sin(2\alpha)}{4} - \frac{\pi}{12} + \frac{\sin(60^\circ) \cdot \cos(2\alpha) + \cos(60^\circ) \cdot \sin(2\alpha)}{4} \right] \\
 &P_R = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{4\pi}{12} - \frac{-\sqrt{3}}{2 \cdot 4} \cdot \cos(2\alpha) - \frac{1}{2 \cdot 4} \cdot \sin(2\alpha) + \frac{\sqrt{3}}{2 \cdot 4} \cos(2\alpha) + \frac{1}{2 \cdot 4} \sin(2\alpha) \right] \\
 1 \quad P_R &= \underline{\underline{\frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{\pi}{3} + \frac{\sqrt{3}}{4} \cos(2\alpha) \right]}}
 \end{aligned}$$

DCM ($30^\circ \leq \alpha < 150^\circ$):

$$\begin{aligned}
 P_R &= \frac{U_{d,RMS}^2}{R} = \frac{\frac{p}{2\pi} \int_{30^\circ+\alpha}^{180^\circ} \hat{u}^2 \cdot \sin^2(\omega t) d\omega t}{R} = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \int_{30^\circ+\alpha}^{180^\circ} \sin^2(\omega t) d\omega t \\
 P_R &= \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{\omega t}{2} - \frac{\sin(2\omega t)}{4} \right]_{30^\circ+\alpha}^{150^\circ+\alpha} = \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{180^\circ}{2} - \frac{\sin(2 \cdot 180^\circ)}{4} - \frac{30^\circ+\alpha}{2} + \frac{\sin(2 \cdot 30^\circ + 2\alpha)}{4} \right] \\
 P_R &= \frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{\pi}{2} - \frac{\sin(360^\circ)}{4} - \frac{\pi}{12} - \frac{\alpha}{2} + \frac{\sin(60^\circ) \cdot \cos(2\alpha) + \cos(60^\circ) \cdot \sin(2\alpha)}{4} \right] \\
 P_R &= \underline{\underline{\frac{3 \cdot \hat{u}^2}{R \cdot 2\pi} \left[\frac{5\pi}{12} - \frac{\alpha}{2} + \frac{\sqrt{3}}{8} \cos(2\alpha) + \frac{1}{8} \sin(2\alpha) \right]}}
 \end{aligned}$$

[4 Credits]

4.4) Reasons for reactive power occurring on the mains side:

For each control angle α the output voltage results in a periodic but non-sinusoidal voltage that contains harmonics. These voltage harmonics lead to current harmonics (independent of the load type) which also appear in the phase currents.

1 → Distortion reactive power D

In addition to that the control angle α also determines the zero crossing of the fundamental phase currents and in consequence influences the phase difference φ between grid-side voltage and current of the same phase.

1 → Fundamental reactive power Q_1 **[2 Credits]**