Marking scheme for the following questions: One point per question if all ticks are correct, half a point if all except one tick are correct, and no points otherwise.

Check **all** answers that apply.

1. Let $x(t) \leftrightarrow X(\omega)$ denote a continuous-time, real valued, periodic signal and its Fourier transform. Then
   - ■ necessarily $X(\omega) = X^*(-\omega)$
   - □ $X(\omega)$ is necessarily real
   - □ $X(\omega)$ cannot be real
   - ■ $X(\omega)$ can be expressed as a (countable) sum of Dirac impulses

2. The energy of a continuous-time signal $x(t)$ is
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$
and its power is
$$P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$$
A signal with positive and finite energy is called an energy signal, a signal with positive and finite power is called a power signal.
   - ■ A signal can never be a power signal and an energy signal at the same time.
   - ■ The unit step signal is a power signal.
   - □ The Dirac $\delta(t)$ impulse is a power signal.
   - ■ A nontrivial periodic signal cannot be an energy signal.

3. The Dirac impulse $\delta(t)$
   - □ satisfies $x(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau$ for every signal $x(t)$
   - ■ satisfies $\delta(at) = \frac{1}{a}\delta(t)$ for all real $a > 0$
   - □ satisfies $\delta(t) * \delta(t) = \delta^2(t)$
   - ■ as the input to an LTI system yields the impulse response $h(t)$ as the output

4. Every LTI system $x(t) \mapsto S\{x(t)\}$ satisfies the following properties:
   - ■ $S\{ax(t)\} = aS\{x(t)\}$ for all $a \in \mathbb{C}$ and all $x(t)$
   - ■ $S\{x(t) + y(t)\} = S\{x(t)\} + S\{y(t)\}$ for all $x(t)$ and all $y(t)$
   - ■ $x(t) \mapsto S\{x(t)\} \implies x(t - T) \mapsto S\{x(t - T)\}$ for all $x(t)$ and any choice of T
   - □ $x(t) \neq y(t) \implies S\{x(t)\} \neq S\{y(t)\}$

5. The Fourier transform of $x(t)$ is $\mathcal{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{j\omega t}dt$. Which of the following are true?
   - □ There exist signals which are time limited (i.e., finite support in the time domain) and at the same time have limited bandwidth.
   - ■ The Fourier transform of a boxcar $x(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$ is $X(\omega) = \text{sinc}\left(\frac{\omega}{2\pi}\right)$.
   - □ The Fourier transform of $x(t) = 1$ is $X(\omega) = 1$.
   - □ The Fourier transform of $x(t) = e^{j\omega_0 t}$ is $X(\omega) = \delta(\omega - \omega_0)$.

6. Let $X(\theta)$ denote the DTFT of a discrete-time signal $x[k]$. The DTFT
   - ■ is linear, i.e., $ax[k] + by[k] \leftrightarrow aX(\theta) + bY(\theta)$.
   - □ the DTFT of $x[k] = 1$ is $X(\theta) = 1$
   - □ is only defined for $\theta \in [-\pi, \pi]$, outside this interval it is zero
   - ■ satisfies $x[k]y[k] \leftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\eta)Y(\theta - \eta)d\eta$.

7. Let $X(\theta) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\theta k}$ denote the DTFT of $x[k]$ given by

$$x[k] = \begin{cases} 1 & k = -3 \\ -1 & k = -2 \\ -2 & k = -1 \\ 4 & k = 0 \\ -2 & k = 1 \\ -1 & k = 2 \\ 1 & k = 3 \\ 0 & \text{otherwise.} \end{cases}$$

   Which of the following statements is true?
   - ■ $X(0) = 0$
   - ■ $X(\theta)$ is real and satisfies $X(\theta) = X(-\theta)$
   - □ $X(\theta)$ is purely imaginary and satisfies $X(\theta) = -X(-\theta)$
   - ■ $X(\pi) = \sum_{k=-3}^{3} x[k]e^{j\pi k} = \sum_{k=-3}^{3} x[k](-1)^k = x[0] = 4$

8. Let $x[k]$ be a finite-length signal and denote by $X[n]$ its DFT.
   - ■ The FFT (Fast Fourier Transform) of $x[k]$ is the same as its DFT.
   - □ One has to perform zero-padding for $x[k]$ and $X[n]$ to have the same length.
   - □ The convolution of finite-length signals corresponds to a circular product of the respective DFTs.
   - ■ $x[k]$ is real if and only if $X[n] = X[(N - n) \mod N]$, where N is the length of $X[n]$.

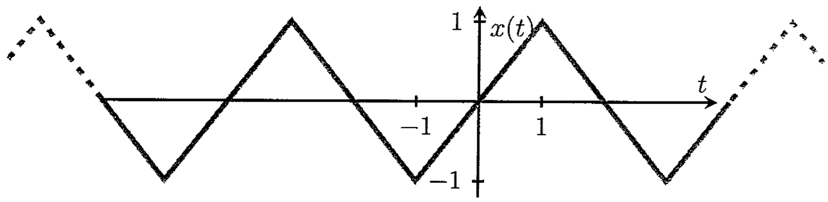
9. The DFT is
   - □ continuous in time
   - ■ periodic in the time domain
   - ■ discrete in the time domain
   - ■ periodic in the frequency domain
   - □ continuous in frequency
   - ■ discrete in the frequency domain

10. Which of the following statements about continuous-time LTI systems are true?
    - ■ A cascade (i.e. feed-forward) connection of two BIBO stable systems must be BIBO stable.
    - ■ The Fourier transform of the impulse response is the frequency response.
    - ■ A causal LTI system has an impulse response with $h(t) = 0$ for $t < 0$.
    - ■ The output $y(t)$ of an LTI system with impulse response $h(t)$ and input $x(t)$ is $y(t) = x(t) * h(t)$, where $*$ denotes convolution.

11. Denote by $X(\omega)$ the Fourier transform of $x(t)$. The signal $x(t)$ is sampled with sampling period $T > 0$ to obtain $x[k] = x(kT)$.

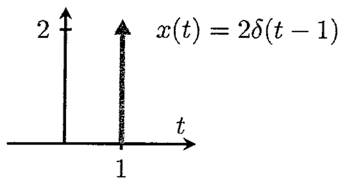
- ☐ If $2\pi/T \geq 2\omega_{\text{max}}$, where ω_{max} is such that $X(\omega) = 0$ for all $|\omega| > \omega_{\text{max}}$, then the original signal $x(t)$ can be perfectly reconstructed.
- ☐ The original signal $x(t)$ can be perfectly reconstructed **only if** $2\pi/T \geq 2\omega_{\text{max}}$.
- ■ Ideal sampling of a time signal as $x[k] = x(kT)$ is not possible in practice, as a measurement is always an average over some (small) time interval ΔT.
- ■ Since a perfect reconstruction filter has a non-causal impulse response $h(t) = \text{sinc}(t/T)$, perfect reconstruction is not possible in real time.

12. Consider the following periodic continuous-time signal and decide which of the properties listed below it satisfies. Let $X(\omega)$ denote its Fourier transform.



- ■ $\text{Re}X(\omega) = 0$
- ☐ $\text{Im}X(\omega) = 0$
- ☐ $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega < \infty$
- ■ $X(\omega) \neq 0$ only for a countable number of frequencies

13. Consider the following continuous-time signal and decide which of the properties listed below it satisfies. Let $X(\omega)$ denote its Fourier transform.



- ☐ $\text{Re}X(\omega) = 0$
- ☐ $\text{Im}X(\omega) = 0$
- ■ There exists a $\phi \in \mathbb{R}$ such that $e^{j\phi\omega}X(\omega)$ is real.
- ■ $X(\omega)$ is periodic.

14. Consider a signal $x[k]$ of length 6. We also know that $|X[0]| = 12$, $|X[1]| = 7$, $|X[2]| = 3$, $|X[3]| = 0$, $|X[4]| = 3$, $|X[5]| = 7$.

- ☐ The signal $x[k]$ must be real valued.
- ☐ The signal $x[k]$ must be purely imaginary.
- ☐ The signal $x[k]$ must be complex valued.
- ■ We are not given enough information to decide for one of the above.

15. Which of the following statements are generally true? (The asterisk $*$ denotes convolution.)

- ■ $x(t) * \delta(t) = x(t)$
- ■ $x(t) * \delta(t - t_0) = x(t - t_0)$
- ☐ $x(t)\delta(t - t_0) = x(t_0)$
- ■ $\delta(t) = 0$ for $t \neq 0$

16. Let $H(s)$ denote the Laplace transformation of the impulse response $h(t)$ of an LTI system.

■ If the Fourier transform of $h(t)$ exists, then it is given by $H(j\omega)$.

□ The impulse response of an LTI system can have either a Laplace transform or a Fourier transform, but not both.

■ The LTI system is BIBO stable if all poles of $H(s)$ lie in the left open complex half-plane.

□ The LTI system is BIBO stable if the Fourier transform of $h(t)$ has no poles for positive frequencies.

Marking scheme for the following questions:

One point per question if all ticks are correct and no points otherwise.

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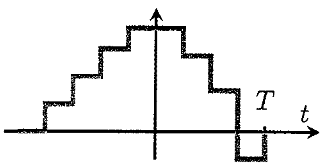
17. The 99% bandwidth B_{99} of a signal $x(t)$ with Fourier transform $X(\omega)$ is defined through the relation

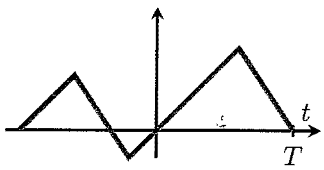
$$\int_{-B_{99}}^{B_{99}} |X(\omega)|^2 d\omega = 0.99 \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

Consider the signals

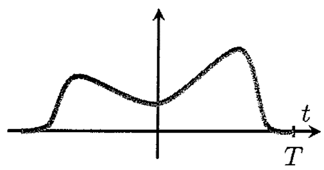
$x(t)$



$y(t)$



$z(t)$



■ Signal $x(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $z(t)$.

□ Signal $x(t)$ has the largest 99% bandwidth, followed by $z(t)$, followed by $y(t)$.

□ Signal $y(t)$ has the largest 99% bandwidth, followed by $x(t)$, followed by $z(t)$.

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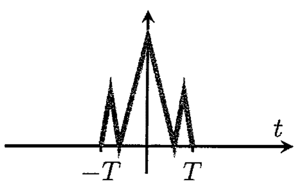
□ Signal $z(t)$ has the largest 99% bandwidth, followed by $x(t)$, followed by $y(t)$.

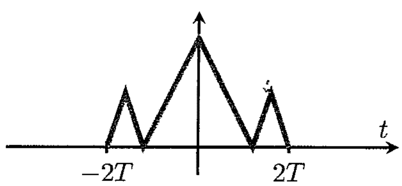
□ Signal $z(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $x(t)$.

18. Consider the signals

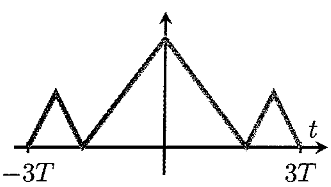
$x(t)$



$y(t)$



$z(t)$



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□ Signal $z(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $x(t)$.

19. The signal $x(t) = e^{0.02t^2}\text{sinc}(t)$ is sampled with a sampling interval $T > 0$. We find that the DTFT of the sampled signal satisfies $X(\theta) = 1$. What is the smallest T that can explain this result?

☐ $T = 1/2$

■ $T = 1$

☐ $T = 2$

☐ There is no such minimal $T > 0$.

20. Let $x[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ -1 & k = 2 \\ 0 & \text{otherwise} \end{cases}$ and $y[k] = \begin{cases} 1 & k = -1 \\ 2 & k = +1 \\ 0 & \text{otherwise.} \end{cases}$

The convolution $z[k] = x[k] * y[k]$ yields

☐ $z[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ 1 & k = 2 \\ 4 & k = 3 \\ -2 & k = 4 \\ 0 & \text{otherwise} \end{cases}$

■ $z[k] = \begin{cases} 1 & k = -1 \\ 2 & k = 0 \\ 1 & k = 1 \\ 4 & k = 2 \\ -2 & k = 3 \\ 0 & \text{otherwise} \end{cases}$

☐ $z[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ 3 & k = 2 \\ 4 & k = 3 \\ 0 & \text{otherwise} \end{cases}$

☐ $z[k] = \begin{cases} 1 & k = -1 \\ 2 & k = 0 \\ 3 & k = 1 \\ 4 & k = 2 \\ 0 & \text{otherwise} \end{cases}$