Simulation of Power Electronic Circuits

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Abstract: Power electronic circuits are characteristically variable structure systems with extreme stiffness due to their switching behaviour. Depending on the point of interest, different models and time scales are used for simulation, but in any case, exact consideration of the switching instants is required. At switching instants, the derivatives of switched signals are not continuous, which must be considered when choosing the integration algorithm and when establishing the pulse width. Last not least, spontaneous switching can be caused by diodes when their currents hit zero. In that case, even the order of the differential equation system changes.

Keywords: power electronic circuits, modeling; simulation

1. Introduction

In power electronic circuits, control and conversion of electrical power is performed by use of semiconductor devices, which are operated in switching mode. When simulating such circuits, several problems are encountered, which result from the switching processes and occur already in basic circuit topologies like a simple buck converter with passive load, see Fig. 1.

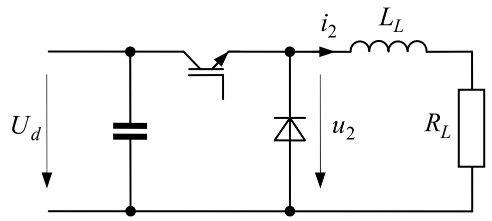

Fig. 1 *Buck converter with passive load*

Like in other circuits, a transistor and a diode form a combined switch connecting the load resistance R_L either to the DC input voltage U_d or to ground. Consequently, the output voltage waveform $u_2(t)$ is of rectangular shape, see Fig. 2, if the input voltage is assumed to be constant and parasitic effects are neglected.

The output current i_2 is also shown in Fig. 2 for continuous conduction mode. During each switching period, i_2 increases and decreases alternatingly depending on the switching state. The ripple of the current is determined by the ratio of time constant $T_L = L_L / R_L$ of the load mesh and of switching period $T_s = 1/f_s$. That ripple represents a disturbance of the desired output current, which is, e. g. in case of a connected electrical

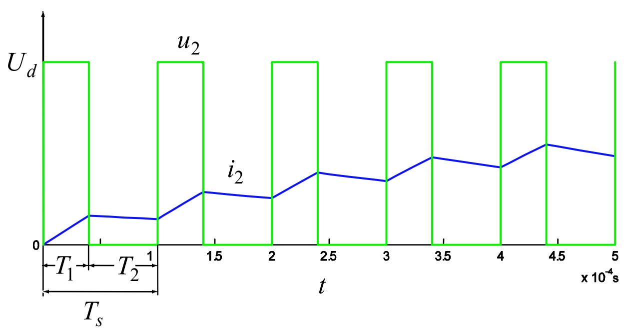

Fig. 2 *Voltage and current of buck converter*

machine, proportional to the torque. Thus, the time ratio T_L / T_s has to be is chosen as large as necessary to make the ripple as small as required by the load.

Under consideration of these conditions, investigation and simulation of power electronic circuits frequently address three possible scopes with different time scales:

- *Averaged interaction of load and power converter*
  With regard to the requirements of the load, the voltage $u_2(t)$ and the current $i_2(t)$ of the power converter should be constant or sinusoidal in case of DC or AC systems, respectively. Thus, the local averages of the output voltage and output current are the quantities of interest. Local average values like $\bar{u}_2(t)$, being variable with regard to time t, are calculated by averaging over one switching period,

$$\bar{u}_2(t) = \int_{t-T_s}^{t} u_2(\tau)\,d\tau \qquad \bar{i}_2(t) = \int_{t-T_s}^{t} i_2(\tau)\,d\tau \qquad (1)$$

Models dealing with such averaged quantities are typically applied, e.g., for investigations of the control behaviour of power electronic systems.

- *Principal switched mode behaviour*
  Within principal investigations of the switching operation mode of the circuit, the power electronic devices are frequently considered as ideal switches without any losses and transient behaviour. However, rough calculations of losses are possible by including a forward resistance and threshold voltage of the devices. The time scale of such investigations depends on the switching frequency and ranges typically between some microseconds and some milliseconds.
- *Commutation and detailed switching behavior*
  The stress of all devices is strongly determined by the switching processes, which cannot be investigated without considering models of the power electronic devices reflecting the switching transients and even the gate drive circuits, considering snubber circuits as well as parasitic inductances and capacitances of the assembly of the power electronic circuitry. Typical time scales are in the range of 10 nanoseconds to microseconds.

With simulation being increasingly used in the power electronic design process in order to cut down design time and costs, general-purpose simulators such as *P-Spice*, *Saber*, *SIMPLORER* are supplying plain up to detailed models, depending on the interest of simulation.

For time-domain simulations over many switching cycles and for optimization tasks requiring repeated simulations, the application of general-purpose tools with complex nonlinear device models may result in excessive computer time and convergence problems. Consequently, this has motivated developments of specialized tools using simplified piecewise linear (PWL) device models [8], facilitating faster simulations at tolerable losses in accuracy. Being aware that this becomes visible during switching transitions, these specialized tools as e.g. *PLECS* should be effectively useful to complement more accurate but computationally less efficient tools.

2. Investigation of principal switched-mode behavior

Transistors and diodes of power electronic circuits are alternating periodically between the blocking and conducting state causing the related branches of the circuit to change between active and passive state. Furthermore, transistor and diode branches are active simultaneously during switch-on and switch-off commutations with additional branches coming into operation, if snubber or clamping circuits are used.

Consequently, the system of differential equations system, which has to be solved at simulation, varies in accordance with the changes of topology and power electric circuits prove to be variable structure systems.

Investigation of the basic behavior of power electronic circuits can be performed efficiently by replacing the semiconductor devices by ideal switches. The according equivalent circuit of the buck converter of Fig. 1 is shown in Fig. 3, while Fig. 4 shows the equivalent circuits with either the transistor, or the diode, or none of these is conducting.

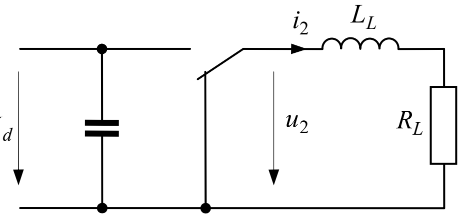

Fig. 3 *Equivalent circuit of buck converter used for investigation of basic behavior*

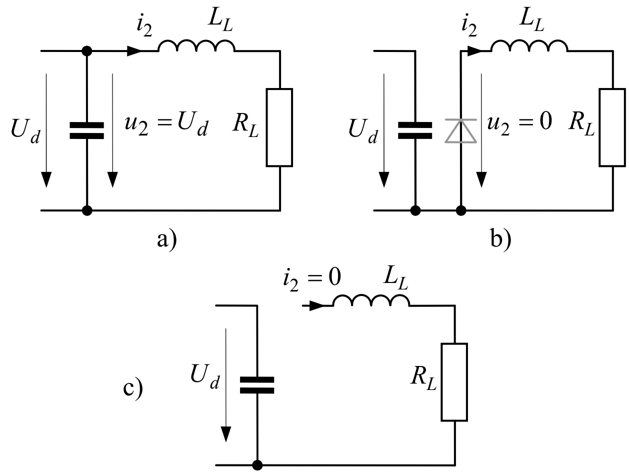

Fig. 4 *Equivalent circuits of buck converter*
a) while transistor is conducting
b) while diode is conducting
c) while both diode and transistor are blocking (discontinuous conduction mode)

As can be seen from eq. (1), the average value of output voltage – hence of the output current – depends strongly on the switching instants. This fact must be considered when performing the integration of the differential equations derived from the equivalent circuits depicted in Fig. 4.

First, it is important to pay attention, when choosing the integration algorithm, because most of the methods like higher order Runge-Kutta assume the derivatives of state variables to be continuous. To handle the discontinuities of the switching behavior, the integration steps must exactly hit the switching instants so that variable step widths are mandatory. Exact consideration of the switching instants is relatively easy, if the switching instants are known in advance, e.g. when a controller has pre-calculated the next time instant, a transistor has to be turned on or off.

In contrast, a diode may become blocking or conducting without any external impact. In the circuit of Fig. 4b, e.g., the diode can become blocking, when its current approaches zero, before the transistor is switched on again, Fig. 4c. In this case, the resulting switching instant results from the evolution of the internal states and must be determined during the simulation run. Additionally, in such cases, the number of conducting branches of the circuit changes so that some of the differential equations change to algebraic ones and vice versa. Note that power electronic components such as thyristors, GTOs and IGBTs operate according to a logical combination of external and internal switching conditions.

Furthermore, the influence of the integration method on the damping of power electronic systems must be considered. Characteristically, power electronic circuits are weakly damped, because losses and thus damping are to be reduced. Hence, damping of oscillations is often implemented only by means of closed-loop control. To consider the success of such a controller during a simulation run, the integration method must represent exactly the natural damping of the actual circuit. The integration method must neither introduce additional numerical damping nor reduce the damping of the system. Unfortunately, a damping shift is common with many integration methods. They have to be carefully selected.

3. Averaged interaction of load and power converter

When simulating the pulsating current, the simulation step width T_{st} must be chosen in accordance with switching period, $T_{st} < T_s$. Switching frequencies of modern switched mode power converters may be up to several 100 kHz over even a MHz, so that maximum simulation steps of a few microseconds or below would result. If the dynamic behavior of interactions between converter and load is to be investigated, typical time constants of which are in the range of some 100 ms or seconds, consequently a huge number of simulation steps would result when simulating e.g. step responses like shown in Fig. 2. Hence, simulation of power electronic circuits may be very time-consuming and large memory resources are required for investigations based on parameter sweeps or sensitivity analysis.

For circuits, which are more complex than the example given above, problems mentioned turn up, too, if the steady state behavior cannot be calculated analytically and shall be investigated by simulation.

To overcome such problems and make simulations less time-consuming and results more transparent, averaging methods have been developed. The basic ideas of the most important methods are explained briefly in the following subsections.

3.1 Averaged modeling of DC systems

The principle of the well known State Space Averaging Method (SPAM) presented in [1] is explained referring to Fig. 5 showing one state variable x within a switching period. In this simple example, the switching period consists of two switching states, with either the transistor or the diode being conducting, if again continuous conduction is assumed.

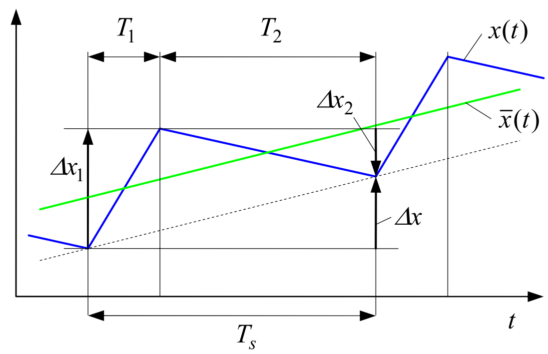

Fig. 5 *State variable during a switching period*

The duration of switching states is determined by the duty cycle $d = T_1 / T_s$ of transistor operation and from the equivalent circuits (as shown for buck converter in Fig. 4) the differential equations can be established as

$$\dot{x} = A_1 x + B_1 u \quad \text{during} \quad T_1 = dTs$$
$$\dot{x} = A_2 x + B_2 u \quad \text{during} \quad T_2 = (1-d)Ts$$

By solving the differential equations, the change Δx of the state variable x within the switching period and the average slope $\dot{\bar{x}}$ of the state variables can be calculated

$$\dot{\bar{x}} \approx \frac{\Delta x}{T_s} = \frac{\Delta x_1 + \Delta x_2}{T_s}$$

This slope is assumed to be the slope of the averaged state variable $\bar{x}$. If linearization is applied repeatedly, calculation of the slope results in

$$\dot{\bar{x}} \approx A(d)\bar{x} + B(d)\bar{u} \tag{2}$$

where

$$\begin{aligned} A(d) &= dA_1 + (1-d)A_2 = A_1 + (A_1 - A_2)d \\ B(d) &= dB_1 + (1-d)B_2 = B_1 + (B_1 - B_2)d \end{aligned} \tag{3}$$

It is not surprising that differential equation (2) proves to be nonlinear because the system matrices are not constant but depend on the duty cycle *d*, which is the control variable used for control of the converter's output voltage and current.

The equivalent circuit of averaged converter model of buck converter shown in Fig. 1 is presented in Fig. 6. A simulation result using the averaged model, compared with the switched-mode simulation, is given in Fig. 7.

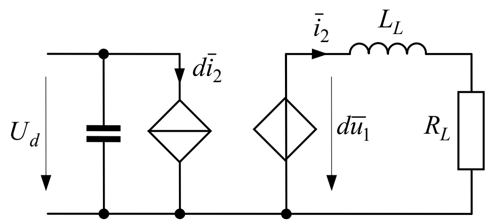

Fig. 6 *Equivalent circuit of averaged buck converter model*

It is important to keep in mind that SPAM is based on several assumptions:

a) The state variables should show low ripples, which is fulfilled, if the switching frequency is constant and high compared to the corner frequency of the low pass filter.

b) No current mode control is involved, otherwise the method needs an enhancement. Assumption a) ensures that the exponential waveforms can be approximated by Taylor Series taking only the DC and first order term into account.

Several improvements of the original SPAM have been suggested in order to investigate converters in the discontinuous conduction mode [2] and to consider fast varying state variables [3] , [4], [5] appearing for example during switching transients, in Quasi Resonant, active clamped or regeneratively snubbed converters.

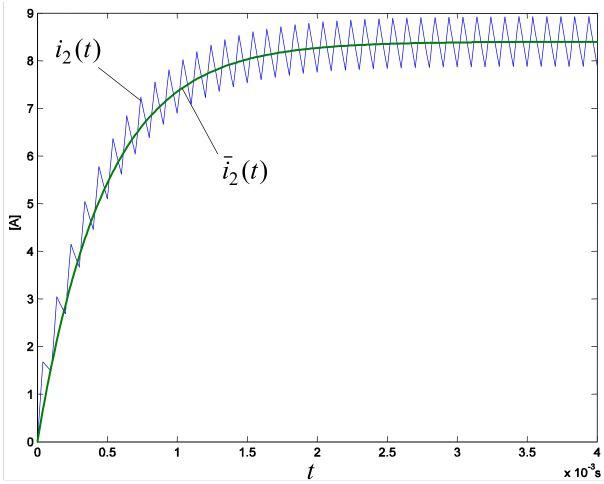

Fig. 7 *Simulation of averaged model vs. switched mode-simulation*

3.2 Averaged modeling for AC systems

A method, which is based on assumption of sinusoidal signals, was suggested in [6]. With this method, all variables are presented by a sine and a cosine oscillation, the amplitudes of which are considered as slowly varying variables. For state variable *x* we get

$$x(t) = x_s(t)\cdot\sin(\omega t) + x_c(t)\cdot\cos(\omega t) \tag{4}$$

From this expression the derivative follows as

$$\begin{aligned} \dot{x}(t) &= \dot{x}_s(t)\cdot\sin(\omega t) + \dot{x}_c(t)\cdot\cos(\omega t) \\ &\quad + \omega x_s(t)\cdot\cos(\omega t) - \omega x_c(t)\cdot\sin(\omega t) \end{aligned} \tag{5}$$

The state variables and their derivatives as well as the sinusoidal fundamental of the input voltage $\bar{u}_1$ exciting the system are inserted into the genuine differential equations of the circuit which results in a new differential equation system consisting of sine and cosine oscillations. Due to the orthogonality of sine and cosine functions, it can be split into one system for the sine waves and another one for the cosine waves. By this measure sine and cosine functions are eliminated from the differential equations and two differential equations systems for state variables xs and xc are obtained.

$$\begin{aligned} \dot{x}_c &= A_{cc}x_c + B_{cc}u_c + C_{sc}x_s \\ \dot{x}_s &= A_{ss}x_s + B_{ss}u_s + C_{cs}x_c \end{aligned} \tag{6}$$

Obviously, both equation systems are coupled by matrices C_{sc}, C_{cs} and can be combined to one system. Alternatively, the cosine and sine amplitudes of one original variable can be considered as components of a complex variable as usual at investigation of three-phase AC systems.

It is important to keep in mind that this method as well as [7] is based on the assumption of sinusoidal signals. Therefore, amplitude and frequency should not vary too quickly and the distortion of waveforms should be low.

4. Investigation of switching and commutation processes

The voltage and current stress as well as the losses of the devices cannot be estimated without investigation of the switching and commutation processes, which proceed very quickly. Due to this fact, inductances of the transistor and diode switch-over mesh termed as commutation inductance cannot be neglected and the problem of a large range of time constants becomes even more severe. Such inductances delay the commutation of the load current between the on-switching branch to the off-switching branch and vice versa, which results in a loss of voltage-seconds on one hand. On the other hand, high peak voltages may appear when switching off the transistor without using a snubber circuit.

In Fig. 8, a buck converter with the frequently used *RCD* clamping circuit (R_s, C_s, D_s) is shown. The circuit diagram also includes the parasitic inductances L_T, L_D and a forward resistance R_F .

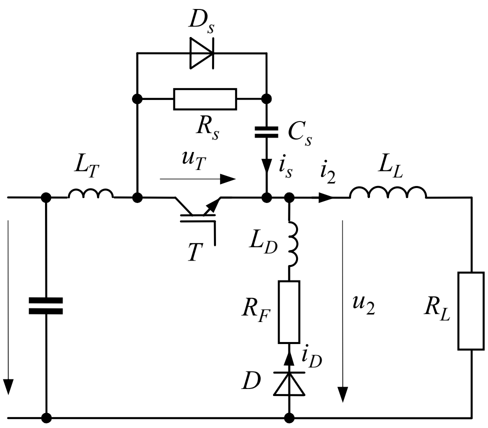

Fig. 8 *Buck converter with clamping circuit and parasitic elements*

At switch-off of transistor, the load current i_2 can commutate rapidly from transistor T to capacitor C_s and diode D_s , because the inductance of the related mesh is very small. During capacitor C_s is being charged, the load current i_2 is commutating to diode D in a resonant process being determined by capacitor C_s and leakage inductances L_T, L_D .

During commutation, an alteration of the output voltage u_2 is observed, see Fig. 9, influencing the local average value. This influence cannot be neglected, if the commutation time T_c is not negligible compared to the switching period. Hence, an adaptation of simulation step width T_{st} is usually required in order to fulfill $T_{st} < T_c < T_s < T_L$.

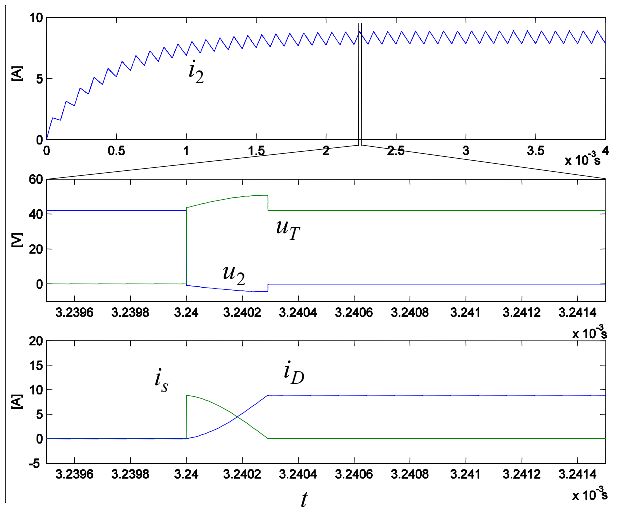

Fig. 9 *Voltages and currents transients during switch-off of the transistor*

5. Conclusions

An introduction to the simulation of power electronic circuits is given in this contribution by explaining function and requirements of switched-mode power converters, which represent variable structure systems and can be controlled by various modulation schemes. It is highlighted that the point of interest decides about time scale, appropriate modeling depth and simulation analysis type. This yields the selected division of the paper by treating the basic converter behavior and the interaction between power converter and the load using different averaging methods depending on the waveform type. Finally modeling enhancements to the averaging are termed, which allow evaluation of switching transients and fast variables in quasi resonant, active clamped or regeneratively snubbed converters.

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Authors' Biography

GROTSTOLLEN, Horst, born in 1938 in Mannheim, Germany, was professor in power electronics and electrical drives at University of Paderborn from 1981 through 2003. He received his Dr.-Ing. (Ph.D.) degree from the Berlin University of Technology in 1972. From 1965 to 1973 he worked at AEG. From 1973 to 1981 he was a senior engineer at the University of Erlangen-Nuremberg, Germany, where he was qualified as a professor by a habilitation on AC servo drives.

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