Pitch Analysis and Control Design for the Linear Motor of a Railway Carriage

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***Abstract* - At the University of Paderborn a mechatronic railway system (NBP)[1] is designed and developed, which is guided by ordinary wheels and rails and is driven via a doubly fed linear motor. The concept is based on the operation of small shuttle units. The drive modules are designed as single axle units. As a result of thrust and normal forces the secondaries can pitch up and down because of the arising torque around the axle. So the pitching angle of the secondary remains an unstable state. Each of them includes two independently fed secondaries, one axle and the respective primary element. This motion should be controlled to ensure a constant airgap between primaries and secondaries. In this paper the modelling of the system and a method for pitch control with state feedback and output integration for flexible mounted linear drives is described. A Kalman filter is used for estimation of the state variables. Finally the simulation and experimental results of pitch control for two secondaries are presented.**

I. Introduction

The NBP railway system comprises autonomous operation of small shuttle units driven by a linear longstatormotor and being fitted with active guidance, suspension and tilting systems. Within the NBP project this railway carriage is designed by using mechatronic design methodology [1] and the shuttle itself is embedded in an overall logistic structure.

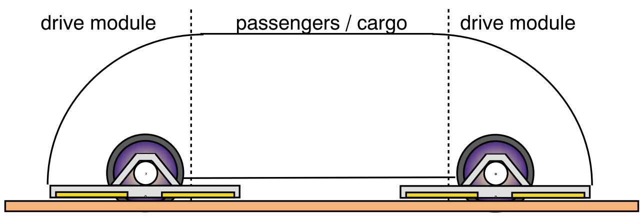

Fig. 1. NBP Shuttle with two linear drive modules

The linear motor is a complex electromechanical system which mainly consists of two components, the primary (longstator), which is installed between the rails, and the secondary (rotor), which is fixed below the undercarriage of the vehicle. From the point of saving energy and improving efficiency, the primaries are divided into many segments that are supplied by different power supply substations. Depending on the position of the carriage the primary segments are switched on accordingly.

As mentioned above the vehicles are small shuttle units which are fitted with two linear drive modules and can drive very flexibly in two directions (Fig.1). In order to meet this requirement the shuttles should be able to accelerate and decelerate at will. Therefore both parts of the linear motor, primary and secondary, are fitted with three phase windings, so that magnetic fields of both primary and secondary, can be orientated at will [2]. The emerging tangential magnetic forces (thrust) between primary and secondary accelerate or brake the vehicle. Because of this kind of feature, the wheels are used only for steering and guiding, the wear will be reduced accordingly.

II. System Modelling

The force producing linear motion between primary and secondary is named thrust force, the force in the plane perpendicular to the direction of thrust is normal force For a rotating machine, the normal forces balance out over its periphery around the airgap. For a linear motor, the normal force cannot be ignored.

Because the drive module consists of one axle, once the total torque on the axle at point O is not zero, the secondaries will pitch along the axle. Then the airgap will be different to the designed value. Not only the thrust force but also the normal force influences the dynamic process and pitch motion of the secondary is regarded. In order to let the airgap remain the designed value, some measures have to be taken to control the total torque and the pitch angle θ to zero.

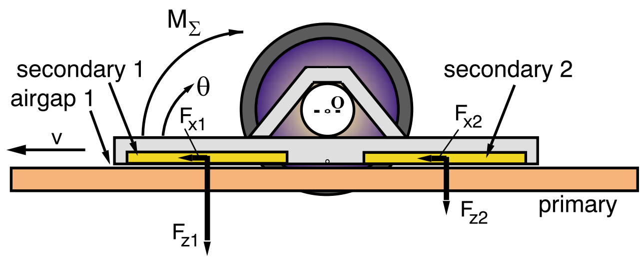

Fig. 2. Linear Drive Module and Coordinate System

Thrust forces and normal forces result in torques on the axle (Fig. 2), which have different directions and values. To compensate this torque by controlling the airgaps on both sides of

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the secondaries, the secondary is divided into two parts, which currents are controlled separately. In this way different currents are supplied in both secondaries, and therefore different normal forces and torques become possible. The generated torques of thrust and normal forces depend on many factors, for example, currents of secondaries, airgap etc.

Obviously, the system is composed of different parts which are mechanically connected to each other. The equations describing the system dynamics can be expressed very efficiently by the use of the Lagrange method. The differential equations that result from use of this method are known as Lagrange's equations[4].

All the forces, which generate a torque on the axle, are displayed in Fig. 3: thrust forces F_x and the vertical forces F_z, excluding the secondary weights. The forces coming from other modules can be neglected for analyzing the system.

The kinetic energy of the system is the sum of the kinetic energy of each mass. The wheel is confined to move in the horizontal direction so its kinetic energy is

$$T_W = (1/2)M_w\dot{X}^2 \tag{1}$$

The secondaries can move in the horizontal and in the vertical direction so their kinetic energy results in

$$T_S = T_{S1} + T_{S2} = \frac{1}{2}m_S(\dot{X}_{S1}^2 + \dot{Z}_{S1}^2) + \frac{1}{2}m_S(\dot{X}_{S2}^2 + \dot{Z}_{S2}^2) \tag{2}$$

with the displacements of secondary 1 and secondary 2

$$X_{S1} = X + \left(d + h\tan\frac{\theta}{2}\right)\cos\theta \tag{3}$$

$$X_{S2} = X - \left(d - h\tan\frac{\theta}{2}\right)\cos\theta \tag{4}$$

$$Z_{S1} = \left(d + h\tan\frac{\theta}{2}\right)\sin\theta \tag{5}$$

$$Z_{S2} = -\left(d - h\tan\frac{\theta}{2}\right)\sin\theta \tag{6}$$

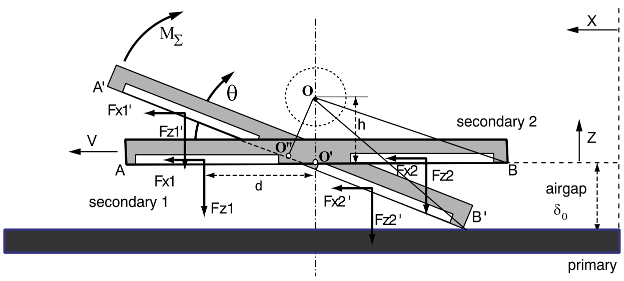

Fig. 3. System Structure with an amplified Airgap

The potential energy V_S is stored in two secondaries. Because of $V_{S1} = -V_{S2}$ the potential energy of the system is zero. Thus the lagrange is

$$L = T - V = T_S + T_W = T_{S1} + T_{S2} + T_W \tag{7}$$

The coordinates are selected as (X, θ), and Lagrangian's equations for this system are

$$\begin{cases} d\dfrac{\partial L}{\partial \dot{X}} - \dfrac{\partial L}{\partial X} = F_{X1} + F_{X2} \\ d\dfrac{\partial L}{\partial \dot{\theta}} - \dfrac{\partial L}{\partial \theta} = (F_{X1} + F_{X2})h + F_{Z2}d\cos\theta - F_{Z1}d\cos\theta \end{cases} \tag{8}$$

Equation (8) is the result of a complex nonlinear model. The system will work near the point: $\theta = 0$, because the maximum pitch angle θ_{max} is smaller than 1 degree and the real angle will be controlled to keep more smaller than θ_{max}. This justifies the approximations: $\tan\theta/2 \approx 0$, $\tan(\theta/2)^2 \approx 0$. Moreover, when θ is very small, the point *O"* is very close to *O'* (Fig. 3).

It is well known that no real system is completely linear, but often the range of operation is such that linearity can be assumed. The stable point of the system is $\theta = 0$ and the real operating point will be close to it. Therefore, (8) can be linearized at this point.

After this approximation, the solution of (8) will result to

$$\begin{cases} \ddot{X} = \dfrac{F_X - m_S h\cos\theta\dot{\theta}}{2m_S + M} \\ \ddot{\theta} = \dfrac{F_X h + \dfrac{(K_1 + K_2 i_{S2}^2)d\cos\theta}{(\delta_0 - d\sin\theta)^2} - \dfrac{(K_1 + K_2 i_{S1}^2)d\cos\theta}{(\delta_0 + d\sin\theta)^2}}{2m_S d^2 + 0.5m_S h^2} \end{cases} \tag{9}$$

and the linearized differential equations from (9)

$$\begin{cases} \Delta\ddot{X} = \dfrac{\Delta F_X}{M + 2m_S} \\ \Delta\ddot{\theta} = a\Delta\theta + b_1\Delta i_{S1} + b_2\Delta i_{S2} + b_3\Delta F_X + b_4\Delta\delta_1 + b_5\Delta\delta_2 = f_0 \end{cases} \tag{10}$$

where

$$F_X = F_{X1} + F_{X2}$$

$$F_{X1} = F_{X2}$$

$$F_X = f(i_{S1}, i_{S2}, i_p, \delta_1, \delta_2)$$

$$F_z = \frac{K_1 + K_2 i_S^2}{\delta^2}$$

$$f_0 = \frac{F_X h + \dfrac{(K_1 + K_2 i_{S2}^2)d\cos\theta}{(\delta_0 - d\sin\theta)^2} - \dfrac{(K_1 + K_2 i_{S1}^2)d\cos\theta}{(\delta_0 + d\sin\theta)^2}}{2m_S d^2 + 0.5m_S h^2}$$

$$a = \left[\frac{\partial f_0}{\partial\theta}\right]_0, b_1 = \left[\frac{\partial f_0}{\partial i_1}\right]_0, b_2 = \left[\frac{\partial f_0}{\partial i_2}\right]_0, b_3 = \left[\frac{\partial f_0}{\partial F_X}\right]_0$$

$b_4 = \left[\frac{\partial f_0}{\partial \delta_1}\right]_0$, $b_5 = \left[\frac{\partial f_0}{\partial \delta_2}\right]_0$

K_1 parameter, related to current of primary

K_2 constant, related to geometric and electric parameters of linear motor

i_S current of secondary

i_{S1} current of secondary 1

i_{S2} current of secondary 2

i_p current of primary

δ airgap between secondary and primary

δ_0 designed value of airgap, namely, reference value

δ_1 airgap between secondary 1 and primary

δ_2 airgap between secondary 2 and primary

$[\,]_0$ certain point at which the system is linearized, that is $\theta = 0$

III. System Control

A. Decoupling of Thrust Control and Pitch Control

Equation (10) represent a fourth-order system with many variables expressed by two differential equations, one for velocity control, the other for pitch control. In order to control the system, it is necessary to adjust the values of some inputs to the system. Normally, some of the inputs will be available for adjustment and these are referred to as the controlled inputs, whereas others will be disturbance inputs which cannot be controlled.

We assume the velocity remains constant under different loads and the airgaps are kept constant at the same time. If the load varies, the thrust force will change accordingly to keep the velocity constant, then it will drive the secondaries away from the balance status, consequently, the airgap between secondary and primary will be changed. And vice visa, because thrust force is related to the currents of the secondaries.

Velocity and pitch motion of secondaries can be controlled independently by decoupled control of F_X and F_Z, in other words, thrust force for velocity control and normal force for pitch control, so that the controllers are easier to design and to implement. Velocity control becomes independent of pitch control if the change of normal forces between secondary and primary doesn't influence the thrust force.

B. Coordinate System

The modelling and the control structure is oriented on a coordinate system, which is common to all secondaries. Here the electrical orientation of the primary current has been chosen as the reference d- axis. Therefore the secondary current in the orthogonal q-axis results in build up of thrust force because of the interaction of stator flux linkage and the orthogonal secondary current vector. The reference of the q-axis current of the primary is set to zero, $i_{Pq} = 0$.

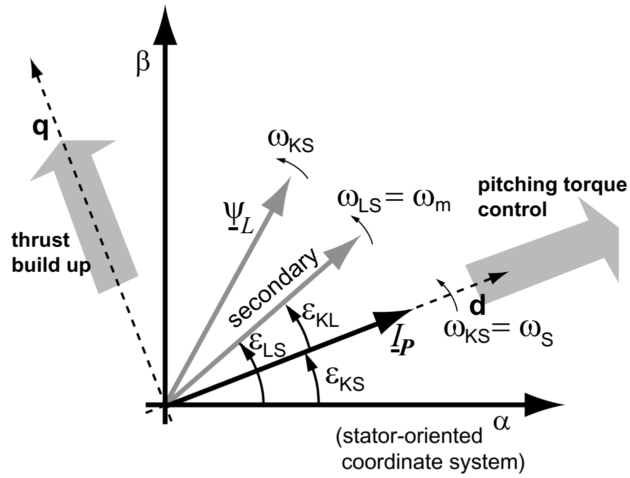

Fig. 4. primary current-oriented coordinate system

So the d-current of secondary i_{Sd} is adjusted to control pitch motion (Fig. 4). Regarding pitching control structure (Fig. 5), thrust force, airgap, etc. can be considered as disturbance inputs. As a result, we only have to analyze the second equation of (10) to control pitch dynamic behaviors.

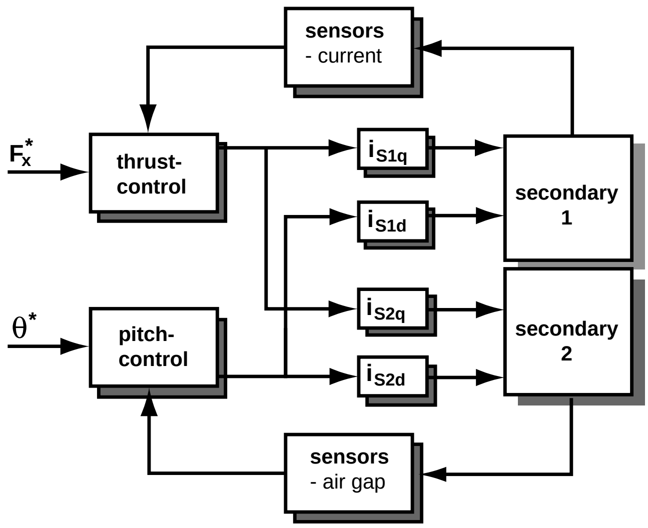

Fig. 5. Control Structure

IV. Closed-loop Control Design

A. Pitching Controller

According to the second equation of (10), the state variables are defined as

$\tilde{X}_1 = \Delta\theta$,

$\tilde{X}_2 = \Delta\dot{\theta}$

inputs and disturbances defined as

$\tilde{U} = (\Delta i_{S1d}, \Delta i_{S2d})'$,

$\tilde{W} = (\Delta F_X, \Delta\delta_1, \Delta\delta_2)'$.

The resulting state-space equations are:

$$\begin{bmatrix}\Delta\dot{\theta}\\ \Delta\ddot{\theta}\end{bmatrix} = \begin{bmatrix}0 & 1\\ a & 0\end{bmatrix}\cdot\begin{bmatrix}\Delta\theta\\ \Delta\dot{\theta}\end{bmatrix} + \begin{bmatrix}0 & 0\\ b_1 & b_2\end{bmatrix}\cdot\begin{bmatrix}\Delta i_{S1d}\\ \Delta i_{S2d}\end{bmatrix}$$

$$+\begin{bmatrix}0 & 0 & 0\\ b_3 & b_4 & b_5\end{bmatrix}\cdot[\Delta F_x \ \Delta\delta_1 \ \Delta\delta_2]$$

$$Y = [1 \quad 0]\cdot[\Delta\theta \quad \Delta\dot{\theta}]' \tag{11}$$

namely

$$\dot{\tilde{X}} = A\tilde{X} + B\tilde{U} + F\tilde{W}$$
$$\tilde{Y} = C\tilde{X} + D\tilde{U} + H\tilde{W}$$

where D=0, H=0. $'\Delta'$ means the relative value to the point at which the system is linearized.

After defining the controlled inputs, a controller is designed which acts on d-axis currents of secondaries Δi_{S1d}, Δi_{S2d}, using the measurements of $\Delta\theta$ and $\Delta\dot{\theta}$. The purpose of the controller is to ensure that $\Delta\theta$ and $\Delta\dot{\theta}$ remain as zero as possible even if the disturbance inputs vary. Thereby, the torque applied on the axle will be zero.

Because the pair [A,B] is completely controllable, a matrix K exists that can give an arbitrary set of eigenvalues of (A-BK). That is, the two roots of the characteristic equation $|\lambda I - A + BK| = 0$ can be arbitrarily placed. We can find feedback controls from the state variables such that the states are driven to zero, namely, the system state $\Delta\theta$ should track the input R(R=0) and the system should suppress undesirable noise and disturbance inputs, no matter what they are. Accordingly, we should introduce integral control in order to have the output track the reference input.

Let the control input u(t) be given by

$$u(t) = -K_1\tilde{X} - K_2\int(R - \tilde{Y})dt \tag{12}$$

where K_1 is a 2×2 feedback-gain matrix, and K_2 is a 2×1 feedback-gain matrix, both with constant elements. Naturally, when K_2=0, the system is a state controller with state feedback.[5]

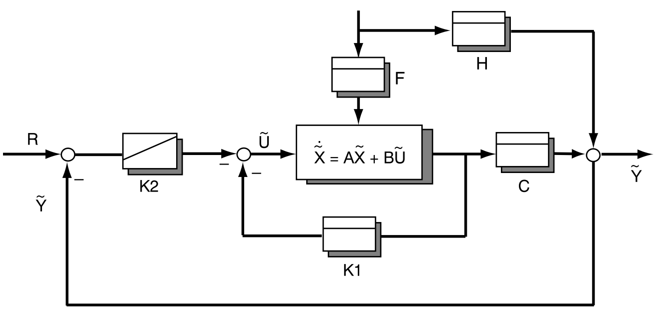

Fig. 6. block diagram of system with state feedback and output integration

Since the feedback system now has one additional integrator (Fig. 5), the overall system is of (2+1)rd order and also controllable. The closed-loop system state equations matrices are

$$\hat{A} = \begin{bmatrix}A & 0\\ \begin{bmatrix}-1 & 0\end{bmatrix} & 0\end{bmatrix} \qquad \hat{B} = \begin{bmatrix}B\\ 0\end{bmatrix} \tag{13}$$

With the integral control, the closed-loop system has three poles to be placed. There are six unknown coefficients of feedback gains, so the solutions are not unique and can be specified. Whether the solutions guarantee closed-loop stability and good response can be tested by the closed-loop poles and the simulation results.

Linear-quadratic controller design is used for continuous-time systems to define the matrix of K_1. As for the elements of matrix K_2, they can be assigned on the basis of the character of system. When $\Delta\theta$ is smaller than the reference input R, namely, $\Delta\theta$-R is negative, $\Delta\theta$ should be increased by reducing the control input Δi_1 or increasing Δi_2. The elements of matrix K_2 have different signs. Since $K_2 = [k_1 \ k_2]'$ is a negative feedback gain, k_1 is positive and k_2 is negative.

B. Steady-State Kalman Filter

Full state feedback with output integration is used to control the pitch motion of secondaries. Only the pitch angle is accessible to measure. The observability test matrix $[C' \quad A'C']$ is with full rank, therefore, the system is also observable. It is possible to design an observer to estimate the pitch velocity that is not directly accessible to measure using the measured pitch angle and all inputs.

The Kalman Filter takes into account measurement noise and process noise, but the optimum Kalman filter is time-varying even if the system is time-invariant and difficult to implement. A suboptimal steady-state filter with a constant Kalman gain matrix L is easier to realize and is usually satisfactory for most applications [3].

The state feedback K and the observer gain L may be designed separately to yield desired closed-loop system behavior and observer behavior. Certainly, the separation principle is available under the controllability and observability assumptions.

V. SIMULATION RESULTS

According to the above design results, the control process of the whole closed-loop system has been simulated by using Matlab/ Simulink.

There are three kinds of disturbance inputs in the linearized system: thrust force, airgap difference, velocity change. During the system simulation, these disturbance inputs vary at different times in order to show which one is critical. In addition, the system is linear one that obeys the principle of superposition. If the separate application of disturbance inputs produces different results, then the simultaneous application of all disturbance inputs will equal to the sum of them.

In Fig. 7 the different influences of three disturbance inputs are shown. At first, a large thrust force affects the system, the overall torque on the axle is above zero and θ will become positive. The normal forces F_{Z1} and F_{Z2} are inverse proportional to the square of the airgaps. When θ is positive, airgap 1 tends to increase and airgap 2 tends to decrease. Correspondingly, F_{Z1} decreases, F_{Z2} increases and the torque M_Σ also increases which will drive the secondaries far from the point we expect. Therefore, the secondary currents step up to counteract the change of airgaps and the torque. Because of the system's inertia, it is impossible to return the steady-state immediately when the disturbance is over. In fact, the system will be stable after a small fluctuation.

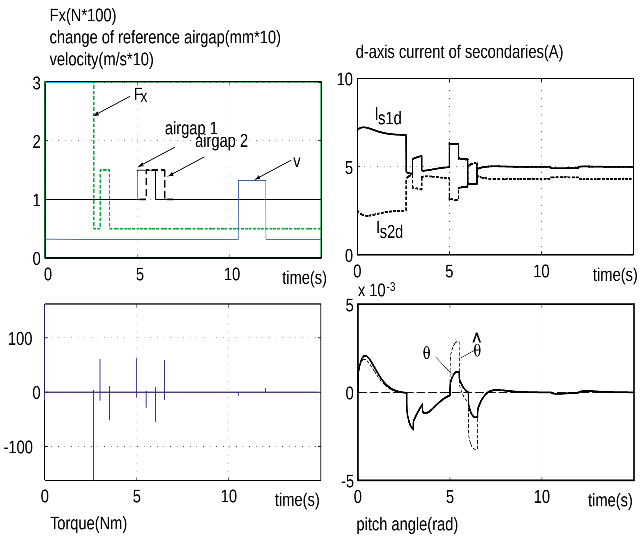

Fig. 7. Simulation Results

VI. Experimental Results

A. *Digital Control and Test Bench*

A test bench for the linear drive module, which is 8 m long, has been built up (Fig. 8). It includes 12 primary elements which are interconnected to two separately fed primary segments. The test vehicle is fitted with two secondary elements, one axle, wheels, airgap sensors, current sensors, position sensor, a DSP-control box, and power supply units. Implementation of control algorithms and analysis of experimental results are done by the host PC, which communicates with the converters via a SSI-interface.

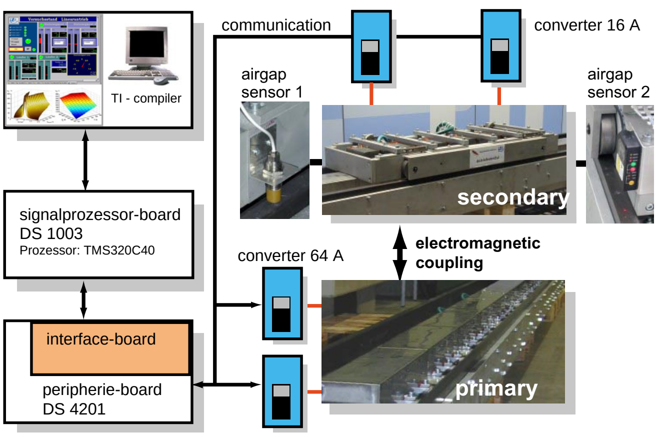

Fig. 8. Structure of the realized test bench

The objective of pitch control is to control two airgaps to a constant value. Therefore, instead of measuring the pitch angle directly, one laser sensor and one eddy current sensor are applied at the two ends of the secondary. Because of the constant radius of the wheel, the pitch angle can be calculated in an easy way via measuring the air gaps on the basis of geometric parameters.

B. *Experimental Results*

Experimental results of pitch control is illustrated in Fig 9. At t = 0, pitch control is disabled, and the pitch angle remains the maximum. Then, the control is turned on at t = 3.3s. After about 0.1s, pitch angle is controlled to the desired value, namely, the airgaps between secondaries and primary segment are kept nearly constant.

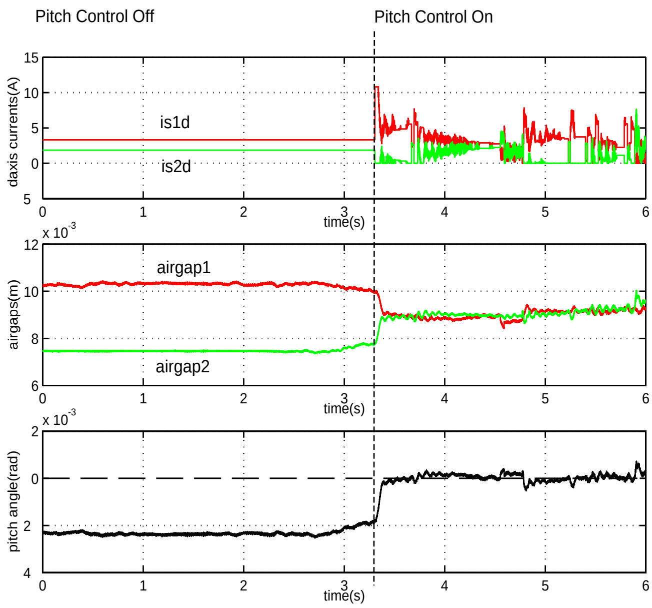

Fig. 9. Experimental Results of Pitch Control

Since the module is a linear drive module, drive control takes precedence over pitch control, in other words, most of secondary current should be reserved for drive control. In addition, if the system is driving and pitch control is on, pitch controller needs only several amperes to operate. As a result, the adjustable range of d-axis current of secondary is limited to [0, 10.8A].

The rails of test bench make up of several steel bars and there are small gaps between them. Two different kind of sensors are applied in order to compare their properties, as a consequence, the measurement error can not be compensated for calculation pitch angle. In addition, mechanical inertia is also existing. All of these factors have influence on the response time and accuracy of pitch control.

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