Marking scheme for the following questions: One point per question if all ticks are correct, half a point if all except one tick are correct, and no points otherwise.

Check all answers that apply.

1. Let $x(t) \leftrightarrow X(\omega)$ denote a continuous-time, real valued, periodic signal and its Fourier transform. Then
   - $X(\omega) = X^*(-\omega)$
   - $X(\omega)$ is necessarily real
   - $X(\omega)$ cannot be real
   - $X(\omega)$ can be expressed as a (countable) sum of Dirac impulses

2. The energy of a continuous-time signal $x(t)$ is
   \[ E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \]
   and its power is
   \[ P_x = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt. \]
   A signal with positive and finite energy is called an energy signal, a signal with positive and finite power is called a power signal.
   - A signal can never be a power signal and an energy signal at the same time.
   - The unit step signal is a power signal.
   - The Dirac $\delta(t)$ impulse is a power signal.
   - A nontrivial periodic signal cannot be an energy signal.

3. The Dirac impulse $\delta(t)$
   - satisfies $x(t) = \int_{-\infty}^{t} x(\tau) \delta(\tau) d\tau$ for every signal $x(t)$
   - satisfies $\delta(at) = \frac{1}{a} \delta(t)$ for all real $a > 0$
   - satisfies $\delta(t) + \delta(t) = \delta(0)$
   - as the input to an LTI system yields the impulse response $h(t)$ as the output

4. Every LTI system $x(t) \rightarrow S\{x(t)\}$ satisfies the following properties:
   - $S\{ax(t)\} = aS\{x(t)\}$ for all $a \in \mathbb{C}$ and all $x(t)$
   - $S\{x(t) + y(t)\} = S\{x(t)\} + S\{y(t)\}$ for all $x(t)$ and all $y(t)$
   - $x(t) \rightarrow S\{x(t)\} \rightarrow x(t - T) \rightarrow S\{x(t - T)\}$ for all $x(t)$ and any choice of $T$
   - $x(t) \neq y(t) \rightarrow S\{x(t)\} \neq S\{y(t)\}$

5. The Fourier transform of $x(t)$ is $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{j\omega t} dt$. Which of the following are true?
   - There exist signals which are time limited (i.e., finite support in the time domain) and at the same time have limited bandwidth.
   - The Fourier transform of a boxcar $x(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & |t| > \frac{1}{2} \end{cases}$ is $X(\omega) = \text{sinc} \left( \frac{\omega}{2\pi} \right)$.
   - The Fourier transform of $x(t) = 1$ is $X(\omega) = 1$.
   - The Fourier transform of $x(t) = e^{j\omega t}$ is $X(\omega) = \delta(\omega - \omega_0)$. 

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6. Let \( X(\theta) \) denote the DTFT of a discrete-time signal \( x[k] \). The DTFT
\[
\begin{align*}
\square \text{ is linear, i.e., } ax[k] + by[k] &\leftrightarrow aX(\theta) + bY(\theta). \\
\square \text{ the DTFT of } x[k] = 1 \text{ is } X(\theta) = 1 \\
\square \text{ is only defined for } \theta \in [-\pi, \pi], \text{ outside this interval it is zero} \\
\square \text{satisfies } x[k]y[k] &\leftrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\eta)Y(\theta - \eta)d\eta.
\end{align*}
\]

7. Let \( X(\theta) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\theta k} \) denote the DTFT of \( x[k] \) given by
\[
x[k] = \begin{cases} 
1 & k = -3 \\
1 & k = -2 \\
-2 & k = -1 \\
4 & k = 0 \\
-2 & k = 1 \\
-1 & k = 2 \\
1 & k = 3 \\
0 & \text{otherwise.}
\end{cases}
\]

Which of the following statements is true?
\[
\begin{align*}
\square X(0) &= 0 \\
\square X(\theta) \text{ is real and satisfies } X(\theta) &= X(-\theta) \\
\square X(\theta) \text{ is purely imaginary and satisfies } X(\theta) &= -X(-\theta) \\
\square X(\pi) &= \sum_{k=-3}^{3} x[k]e^{j\pi k} = \sum_{k=-3}^{3} x[k](-1)^k = x[0] = 4
\end{align*}
\]

8. Let \( x[k] \) be a finite-length signal and denote by \( X[n] \) its DFT.
\[
\begin{align*}
\square \text{ The FFT (Fast Fourier Transform) of } x[k] \text{ is the same as its DFT.} \\
\square \text{ One has to perform zero-padding for } x[k] \text{ and } X[n] \text{ to have the same length.} \\
\square \text{ The convolution of finite-length signals corresponds to a circular product of the respective DFTs.} \\
\square x[k] \text{ is real if and only if } X[n] = X[(N - n) \mod N], \text{ where } N \text{ is the length of } X[n].
\end{align*}
\]

9. The DFT is
\[
\begin{align*}
\square \text{ continuous in time} \\
\square \text{ periodic in the time domain} \\
\square \text{ discrete in the time domain} \\
\square \text{ periodic in the frequency domain} \\
\square \text{ continuous in frequency} \\
\square \text{ discrete in the frequency domain}
\end{align*}
\]

10. Which of the following statements about continuous-time LTI systems are true?
\[
\begin{align*}
\square \text{ A cascade (i.e. feed-forward) connection of two BIBO stable systems must be BIBO stable.} \\
\square \text{ The Fourier transform of the impulse response is the frequency response.} \\
\square \text{ A causal LTI system has an impulse response with } h(t) = 0 \text{ for } t < 0. \\
\square \text{ The output } y(t) \text{ of an LTI system with impulse response } h(t) \text{ and input } x(t) \text{ is } y(t) = x(t) * h(t), \text{ where } * \text{ denotes convolution.}
\end{align*}
\]
11. Denote by $X(\omega)$ the Fourier transform of $x(t)$. The signal $x(t)$ is sampled with sampling period $T > 0$ to obtain $x[k] = x(kT)$.

- If $2\pi/T \geq 2\omega_{\text{max}}$, where $\omega_{\text{max}}$ is such that $X(\omega) = 0$ for all $|\omega| > \omega_{\text{max}}$, then the original signal $x(t)$ can be perfectly reconstructed.

- The original signal $x(t)$ can be perfectly reconstructed only if $2\pi/T \geq 2\omega_{\text{max}}$.

- Ideal sampling of a time signal as $x[k] = x(kT)$ is not possible in practice, as a measurement is always an average over some (small) time interval $\Delta T$.

- Since a perfect reconstruction filter has a non-causal impulse response $h(t) = \text{sinc}(t/T)$, perfect reconstruction is not possible in real time.

12. Consider the following periodic continuous-time signal and decide which of the properties listed below it satisfies. Let $X(\omega)$ denote its Fourier transform.

- $\text{Re}X(\omega) = 0$
- $\text{Im}X(\omega) = 0$
- $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega < \infty$
- $X(\omega) \neq 0$ only for a countable number of frequencies

13. Consider the following continuous-time signal and decide which of the properties listed below it satisfies. Let $X(\omega)$ denote its Fourier transform.

- $\text{Re}X(\omega) = 0$
- $\text{Im}X(\omega) = 0$
- There exists a $\phi \in \mathbb{R}$ such that $e^{i\phi \omega} X(\omega)$ is real.
- $X(\omega)$ is periodic.


- The signal $x[k]$ must be real valued.
- The signal $x[k]$ must be purely imaginary.
- The signal $x[k]$ must be complex valued.
- We are not given enough information to decide for one of the above.

15. Which of the following statements are generally true? (The asterisk * denotes convolution.)

- $x(t) * \delta(t) = x(t)$
- $x(t) * \delta(t - t_0) = x(t - t_0)$
- $x(t)\delta(t - t_0) = x(t_0)$
- $\delta(t) = 0$ for $t \neq 0$
16. Let $H(s)$ denote the Laplace transformation of the impulse response $h(t)$ of an LTI system.

- If the Fourier transform of $h(t)$ exists, then it is given by $H(j\omega)$.
- The impulse response of an LTI system can have either a Laplace transform or a Fourier transform, but not both.
- The LTI system is BIBO stable if all poles of $H(s)$ lie in the left open complex half-plane.
- The LTI system is BIBO stable if the Fourier transform of $h(t)$ has no poles for positive frequencies.

Marking scheme for the following questions:
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17. The 99% bandwidth $B_{99}$ of a signal $x(t)$ with Fourier transform $X(\omega)$ is defined through the relation

$$\int_{-B_{99}}^{B_{99}} |X(\omega)|^2 d\omega = 0.99 \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega.$$

Consider the signals

\[ x(t) \quad y(t) \quad z(t) \]

- Signal $x(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $z(t)$.
- Signal $x(t)$ has the largest 99% bandwidth, followed by $z(t)$, followed by $y(t)$.
- Signal $y(t)$ has the largest 99% bandwidth, followed by $x(t)$, followed by $z(t)$.
- Signal $y(t)$ has the largest 99% bandwidth, followed by $z(t)$, followed by $x(t)$.
- Signal $z(t)$ has the largest 99% bandwidth, followed by $x(t)$, followed by $y(t)$.
- Signal $z(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $x(t)$.

18. Consider the signals

\[ x(t) \quad y(t) \quad z(t) \]

- Signal $x(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $z(t)$.
- Signal $x(t)$ has the largest 99% bandwidth, followed by $z(t)$, followed by $y(t)$.
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- Signal $y(t)$ has the largest 99% bandwidth, followed by $z(t)$, followed by $x(t)$.
- Signal $z(t)$ has the largest 99% bandwidth, followed by $x(t)$, followed by $y(t)$.
- Signal $z(t)$ has the largest 99% bandwidth, followed by $y(t)$, followed by $x(t)$.
19. The signal $x(t) = e^{0.02t^3} \text{sinc}(t)$ is sampled with a sampling interval $T > 0$. We find that the DTFT of the sampled signal satisfies $X(\theta) = 1$. What is the smallest $T$ that can explain this result?

- $T = 1/2$
- $T = 1$
- $T = 2$
- There is no such minimal $T > 0$.

20. Let $x[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ -1 & k = 2 \\ 0 & \text{otherwise} \end{cases}$ and $y[k] = \begin{cases} 1 & k = -1 \\ 2 & k = +1 \\ 0 & \text{otherwise} \end{cases}$.

The convolution $z[k] = x[k] * y[k]$ yields

- $z[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ 1 & k = 2 \\ 4 & k = 3 \\ -2 & k = 4 \\ 0 & \text{otherwise} \end{cases}$

- $z[k] = \begin{cases} 1 & k = -1 \\ 2 & k = 0 \\ 1 & k = 1 \\ 4 & k = 2 \\ -2 & k = 3 \\ 0 & \text{otherwise} \end{cases}$

- $z[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 1 \\ 3 & k = 2 \\ 4 & k = 3 \\ 0 & \text{otherwise} \end{cases}$

- $z[k] = \begin{cases} 1 & k = -1 \\ 2 & k = 0 \\ 3 & k = 1 \\ 4 & k = 2 \\ 0 & \text{otherwise} \end{cases}$