The following questions cover a range of topics you should master (i.e. >75% correct answers) before starting the course Fields&Waves in the Electrical Systems Engineering program at Paderborn University:

1. Evaluate
   (a) \( \sin \frac{\pi}{2} = 1 \)
   (b) \( \cos \frac{\pi}{2} = 0 \)
   (c) \( \sin^2 x + \cos^2 x = 1 \)
   (d) \( \exp(0) = 1 \)
   (e) \( \exp(-\frac{\pi}{2} j) = -j \)

2. Express \( e^{jx} \) in terms of \( \sin \) and \( \cos \) (Euler’s identity):
   \( e^{jx} = \cos x + j \sin x \)

3. Give the general real-valued solution of the ODEs
   (a) \( \frac{d^2}{dt^2} y(t) = -\omega^2 y(t) \) (with \( \omega \neq 0 \))
   \( y(t) = a \cos(\omega t) + b \sin(\omega t) \) or \( a \sin(\omega t + \phi) \)
   (b) \( \frac{d}{dt} y(t) = -\gamma y(t) \) (with \( \gamma \neq 0 \))
   \( y(t) = ae^{-\gamma t} \)

4. Give the solution of the Fourier integral \( g(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \) for
   (a) \( g(t) = \frac{d}{dt} f(t) \) (assume \( f(\omega) \) is known): \( g(\omega) = -j\omega f(\omega) \)
   (b) \( g(t) = f(t) e^{j\omega_0 t} \) (assume \( f(\omega) \) is known): \( g(\omega) = f(\omega - \omega_0) \)
   (c) \( g(t) = \sin(\omega_0 t) \) \( g(\omega) = \pi \delta(\omega - \omega_0) - \pi \delta(\omega + \omega_0) \)

5. Vector products, Give
   (a) the projection of a vector \( \vec{a} \) on a normalized vector \( \vec{n} \) : \( \vec{a} \cdot \vec{n} = a \cos \phi \)
   (b) the inner product \( \vec{a} \cdot \vec{b} \) in cartesian coordinates: \( = axbx + ayby + azbz \)
   (c) the length of a vector \( \vec{a} \) using the inner product: \( = \sqrt{\vec{a} \cdot \vec{a}} \)
   (d) the vector product \( \vec{a} \times \vec{b} \) in cartesian coordinates: \( = \begin{pmatrix} aybz - azby \\ azbx - axbz \\ axby - aybx \end{pmatrix} \)

6. Evaluate the following expressions (or mark if invalid):
   (a) \( \text{grad} 5 = 0 \)
   (b) \( \text{curl} 4 = \text{Invalid expression} \)
   (c) \( \text{grad}(x^2 + y^3) = \begin{pmatrix} 2x \\ 3y^2 \\ 0 \end{pmatrix} \)
   (d) \( \text{curl \ grad} \ \vec{v}(\vec{r}) = 0 \)
(e) \[ \text{div} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \]

(f) \[ \text{curl} \begin{pmatrix} 0 \\ 0 \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \]

7. State Stokes’ and Gauss’ theorems:
   
   (a) \[ \int_V \text{div} \vec{v}(\vec{r}) \, dV = \int_{\partial V} \vec{v}(\vec{r}) \cdot d\vec{a} \]
   
   (b) \[ \int_A \text{curl} \vec{v}(\vec{r}) \cdot d\vec{a} = \int_{\partial A} \vec{v}(\vec{r}) \cdot d\vec{s} \]

8. Give the electrostatic potential of a point charge \( q \) located at the position \( \vec{s} \):
   \[ \phi(\vec{r}) = \frac{q}{4\pi\epsilon_0|\vec{r} - \vec{s}|} \]

9. Write down the four Maxwell equations (for material/medium, in differential form, SI units):
   
   (a) \[ \text{curl} \vec{E} = -\frac{d}{dt} \vec{B} \]
   
   (b) \[ \text{curl} \vec{H} = \frac{d}{dt} \vec{D} + \vec{J} \]
   
   (c) \[ \text{div} \vec{D} = \rho \]
   
   (d) \[ \text{div} \vec{B} = 0 \]

10. Which electric and magnetic field components are continuous at an interface? B normal, E tangential

11. For a perfect electric conductor, the electric field strength
   
   (a) inside is: 0
   
   (b) at the surface is: orthogonal on surface

12. In a medium give (in terms of the real-valued e.m. fields) the definitions of
   
   (a) the Poynting vector: \( \vec{S} = \vec{E} \times \vec{H} \)
   
   (b) the electromagnetic energy (in a volume \( V \)): \( W = \frac{1}{2} \int_V (\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) \, dV \)

13. Give the units (in SI) of
   
   (a) the electric field strength: \( [\vec{E}] = \text{V/m} \)
   
   (b) the magnetic flux density: \( [\vec{B}] = \text{Vs/m}^2 = \text{T} \)
   
   (c) the current density: \( [\vec{J}] = \text{A/m}^2 \)
   
   (d) the charge density: \( [\rho] = \text{C/m}^3 = \text{As/m}^3 \)